2014 Autumn Semester, course for graduate student
Lecture notes: Physics of Laser-Plasma Interaction

VI. Parametric instabilities in underdense plasma
（次临界密度等离子体中的参量不稳定性）

Bin Qiao

School of Physics

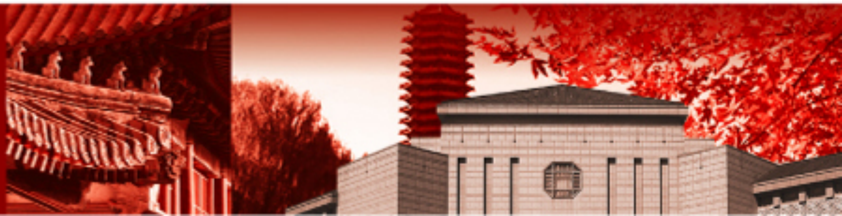
Peking University, Beijing, P. R. China

Email: bqiao@pku.edu.cn

Office: Room 544 (South), Physics Building

Tel: 62745005

北京大學



Coupling between laser and the plasma oscillators

表-1　激光等离子体中的主要相互作用过程

不稳定过程	能量守恒	发生区域	后果
共振吸收	$\omega_0 = \omega_{epw}$	$\sim n_{cr}$	超热电子
受激 Brillouin 散射	$\omega_0 = \omega_s + \omega_{isw}$	$< n_{cr}$	减少激光能量沉积
受激 Raman 散射	$\omega_0 = \omega_s + \omega_{epw}$	$\leq n_{cr}/4$	超热电子，减少激光能量沉积
双等离子体衰变	$\omega_0 = \omega_{epw} + \omega_{epw}$	$\sim n_{cr}/4$	超热电子
离子声衰变	$\omega_0 = \omega_{isw} + \omega_{epw}$	$\sim n_{cr}$	超热电子

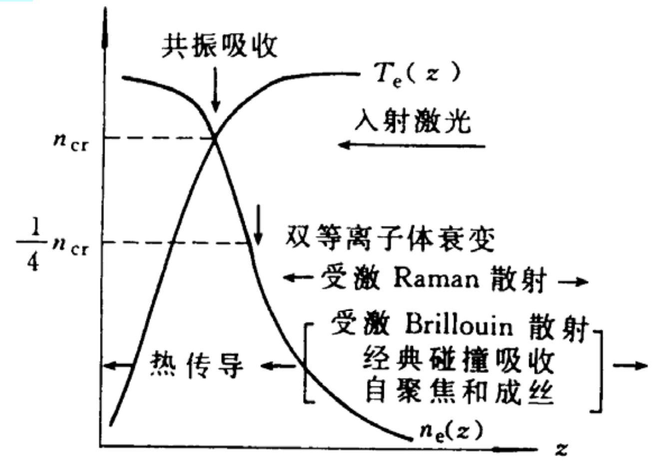


图 2.3　激光等离子体主要耦合过程发生的温度密度区

为了实现高增益靶丸内爆，必须最大限度地限制超热电子的产生。

$$\omega_l = \omega_1 + \omega_2$$
$$k_l = k_1 + k_2$$

Manley-Rowe 关系式

$$\omega_{ek}^2 = \omega_{pe}^2 + 3k_{ek}^2 v_e^2$$

$$\omega_0^2 = \omega_{pe}^2 + k_0^2 c^2$$

$$\omega_{is}^2 = k_{is}^2 C_s^2$$

激光强度：10^{14}-10^{16} W/cm^2

Outline

VI.1 受激Raman散射

(Stimulated Raman Scattering)

VI.2 双等离子体衰变

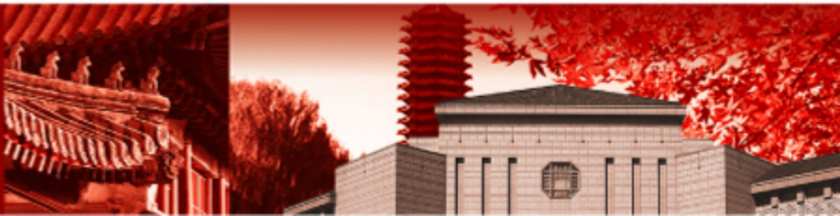
(Two Plasma Decay)

VI.3 受激Brillouin散射

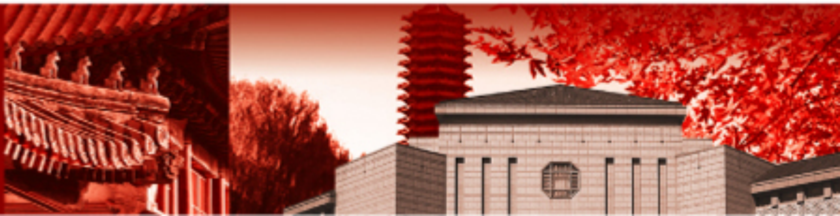
(Stimulated Brillouin Scattering)

VI.4 激光的自聚焦与成丝不稳定性

(Self-focusing and filamentation instability)



VI.2 双等离子体衰变
（Two Plasma Decay）



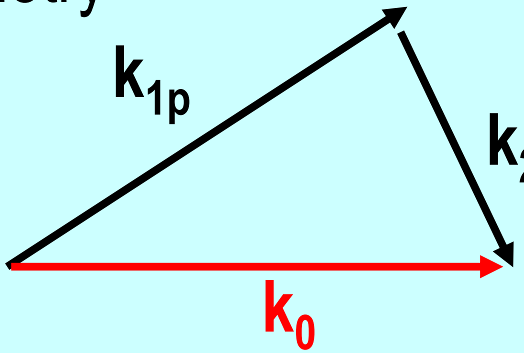
(1) 什么是双等离子体衰变?

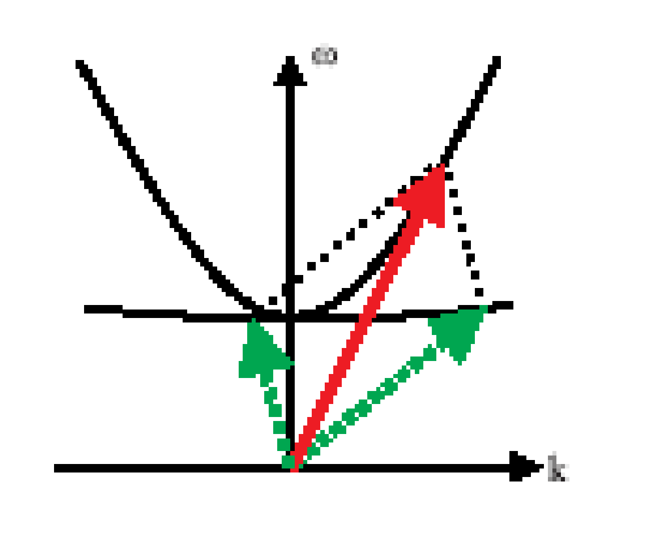
- A laser light decays into two plasma waves:

$$\omega_0 = \omega_{e1} + \omega_{e2}\ , \vec{k}_0 = \vec{k}_1 + \vec{k}_2$$

$$\omega_{e1} \sim \omega_{e2} \sim \omega_{pe} \to \omega_0 \sim 2\omega_{pe}\ (n \sim \frac{n_{cr}}{4}) \to |k_0| \sim \frac{\sqrt{3}}{2}\frac{\omega_0}{c}$$

- Phase match conditions can be realized only in 2 or 3 dimension geometry





（2）双等离子体衰变的色散关系

离子：固定不动的电中性背景，电子：电子流体

$$\vec{u}_e=\vec{u}_L'+\vec{v}_{os},\vec{v}_{os}=\frac{e\vec{A}_0}{m_e c}$$

连续性方程和运动方程

$$\frac{\partial n_e}{\partial t}+\nabla\cdot(n_e\vec{u}_e)=0.$$

$$\frac{\partial \vec{u}_L}{\partial t}=\frac{e}{m_e}\nabla\Phi-\frac{1}{2}\nabla(\vec{u}_L+\frac{e\vec{A}}{m_e c})^2-\frac{\nabla P_e}{m_e n_e}.$$

取绝热状态方程

线性化 $n_e=n_0+n_e',\quad \vec{u}_L=\vec{u}',\vec{A}=\vec{\mathrm{A}}_L+\vec{A}',\Phi=\Phi'$

$\frac{\partial}{\partial t}\to$

$$\frac{\partial n_e'}{\partial t}+n_0\nabla\cdot\vec{u}_L'+\vec{v}_{os}\cdot\nabla n_e'=0.$$

$\vec{u}_L',n_e',\Phi'$ 均是无穷小量

$\nabla\cdot\to$

$$\frac{\partial \vec{u}_L'}{\partial t}=\frac{e}{m_e}\nabla\Phi'-\frac{3v_e^2}{n_0}\nabla n_e'-\nabla(\vec{v}_{os}\cdot\vec{u}_L').$$

（2）双等离子体衰变的色散关系

$$(\frac{\partial^2}{\partial t^2}+\omega_{pe}{}^2-3v_e{}^2\nabla^2)n_e'+\frac{\partial}{\partial t}(\vec{v}_{os}\cdot\nabla n_e')-n_0\nabla^2(\vec{v}_{os}\cdot\vec{u}_L')=0$$

$$v_{os}=\frac{v_{os}}{2}[\exp(i\vec{k}_0\cdot\vec{x}-i\omega_0 t)+\exp(-i\vec{k}_0\cdot\vec{x}+i\omega_0 t)],$$

$$n_e'\sim\exp(i\vec{k}\cdot\vec{x}-i\omega t),$$

$$u_L'\sim\exp(i\vec{k}\cdot\vec{x}-i\omega t)$$

$$(-\omega^2+\omega_{ek}{}^2)n_e'(\vec{k},\omega)+\frac{\omega}{2}\vec{k}\cdot\vec{v}_{os}[n_e'(\vec{k}-\vec{k}_0,\omega-\omega_0)+n_e'(\vec{k}+\vec{k}_0,\omega+\omega_0)]+$$

$$\frac{n_0k^2}{2}\vec{v}_{os}\cdot[\vec{u}_L'(\vec{k}-\vec{k}_0,\omega-\omega_0)+\vec{u}_L'(\vec{k}+\vec{k}_0,\omega+\omega_0)]=0.$$

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$$\vec{u}_L'(\vec{k},\omega)\cong\frac{\vec{k}}{k^2}\omega\frac{n_e'(\vec{k},\omega)}{n_0}$$

$\Rightarrow$

$$n_0k^2\vec{u}_L'(\vec{k},\omega)\cong\vec{k}\omega n_e'(\vec{k},\omega)$$

（2）双等离子体衰变的色散关系

$\omega-\omega_0\approx-\omega_{pe}$

取ω～ ω_{pe}，并略去 $\omega+\omega_0$ 或 $\omega-2\omega_0$ 远离共振的任何响应，注意 $\vec{k}_0\cdot\vec{v}_{os}=0$

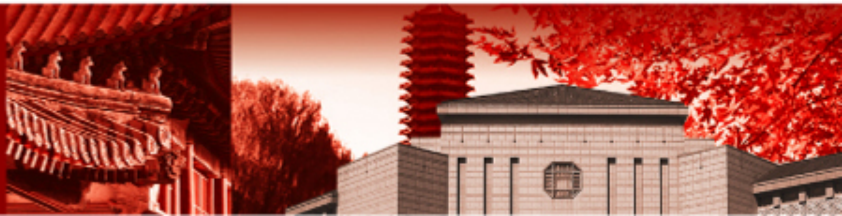
得到 $n_e'(\vec{k},\omega)$ 和 $n_e'(\vec{k}-\vec{k}_0,\omega-\omega_0)$ 的耦合方程

$$(-\omega^2+\omega_{ek}{}^2)n_e'(\vec{k},\omega)+\frac{\vec{v}_{os}}{2}\cdot[\omega\vec{k}n_e'(\vec{k}-\vec{k}_0,\omega-\omega_0)+n_0k^2\vec{u}_L'(\vec{k}-\vec{k}_0,\omega-\omega_0)]=0.$$

$$[-(\omega-\omega_0)^2+\omega_{ek-k_0}{}^2]n_e'(\vec{k}-\vec{k}_0,\omega-\omega_0)+\frac{\vec{v}_{os}}{2}\cdot[(\omega-\omega_0)\vec{k}n_e'(\vec{k},\omega)+n_0(\vec{k}-\vec{k}_0)^2\vec{u}_L'(\vec{k},\omega)]=0.$$

这些方程描述了具有波数**k**和**k-k$_0$**的电子等离子体波通过激光的耦合。

$$(-\omega^2+\omega_{ek}{}^2)n_e'(\vec{k},\omega)+\frac{\omega}{2}\vec{k}\cdot\vec{v}_{os}[n_e'(\vec{k}-\vec{k}_0,\omega-\omega_0)+n_e'(\vec{k}+\vec{k}_0,\omega+\omega_0)]+$$
$$\frac{n_0k^2}{2}\vec{v}_{os}\cdot[\vec{u}_L'(\vec{k}-\vec{k}_0,\omega-\omega_0)+\vec{u}_L'(\vec{k}+\vec{k}_0,\omega+\omega_0)]=0.$$



（2）双等离子体衰变的色散关系

Fourier分析连续性方程 $\frac{\partial n_e'}{\partial t}+n_0\nabla\cdot\vec{u}_L'+\vec{v}_{os}\cdot\nabla n_e'=0.$

$\omega_0=\omega_{e1}+\omega_{e2}\ ,\vec{\mathrm{k}}_0=\vec{\mathrm{k}}_1+\vec{\mathrm{k}}_2$

用近似

$$\vec{u}_L'(\vec{k},)\cong\frac{\vec{k}}{k^2}\omega\frac{n_e'(\vec{k},\omega)}{n_0}$$

$(\omega\approx\omega_{pe}$和$\omega-\omega_0\approx-\omega_{pe})$

$$\vec{u}_L'(\vec{k}-\vec{k}_0,\omega-\omega_0)\cong\frac{\vec{k}-\vec{k}_0}{|k-\vec{k}_0|^2}\omega\frac{n_e'(\vec{k}-\vec{k}_0,\omega-\omega_0)}{n_0}$$

消去

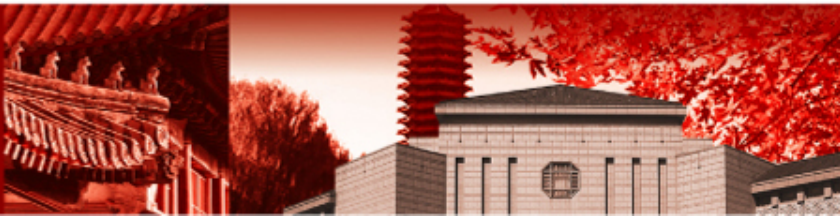
$\vec{u}_L'(\vec{k},\omega)$，$\vec{u}_L'(\vec{k}-\vec{k}_0,\omega-\omega_0)$

$$(-\omega^2+\omega_{ek}{}^2)n_e'(\vec{k},\omega)+\frac{\vec{v}_{os}}{2}\cdot[\omega\vec{k}n_e'(\vec{k}-\vec{k}_0,\omega-\omega_0)+n_0k^2\vec{u}_L'(\vec{k}-\vec{k}_0,\omega-\omega_0)]=0.$$

$$[-(\omega-\omega_0)^2+\omega_{ek-k_0}{}^2]n_e'(\vec{k}-\vec{k}_0,\omega-\omega_0)+\frac{\vec{v}_{os}}{2}\cdot[(\omega-\omega_0)\vec{k}n_e'(\vec{k},\omega)+n_0(\vec{k}-\vec{k}_0)^2\vec{u}_L'(\vec{k},\omega)]=0.$$

得到色散关系

$$(\omega^2-\omega_{ek}{}^2)[(\omega-\omega_0)^2-\omega_{ek-k_0}{}^2]=\frac{\vec{k}\cdot\vec{v}_{os}\omega_{pe}[(\vec{k}-\vec{k}_0)^2-k^2]}{2k|\vec{k}-\vec{k}_0|}$$



（2）双等离子体衰变的增长率

讨论：

用 $\omega = \omega_r + i\gamma$ 代入并利用频率匹配条件，可以求出增长率

$$\gamma = \frac{\vec{k}\cdot\vec{v}_{os}}{4}\left|\frac{(\vec{k}-\vec{k}_0)^2 - k^2}{k|\vec{k}-\vec{k}_0|}\right|.$$

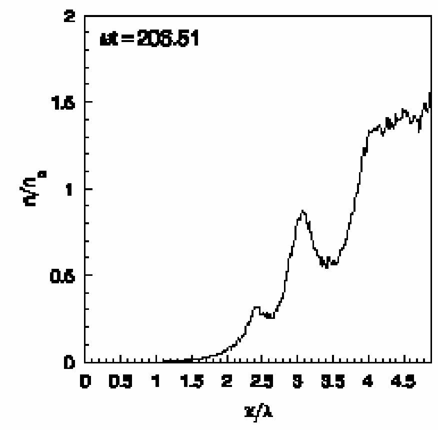
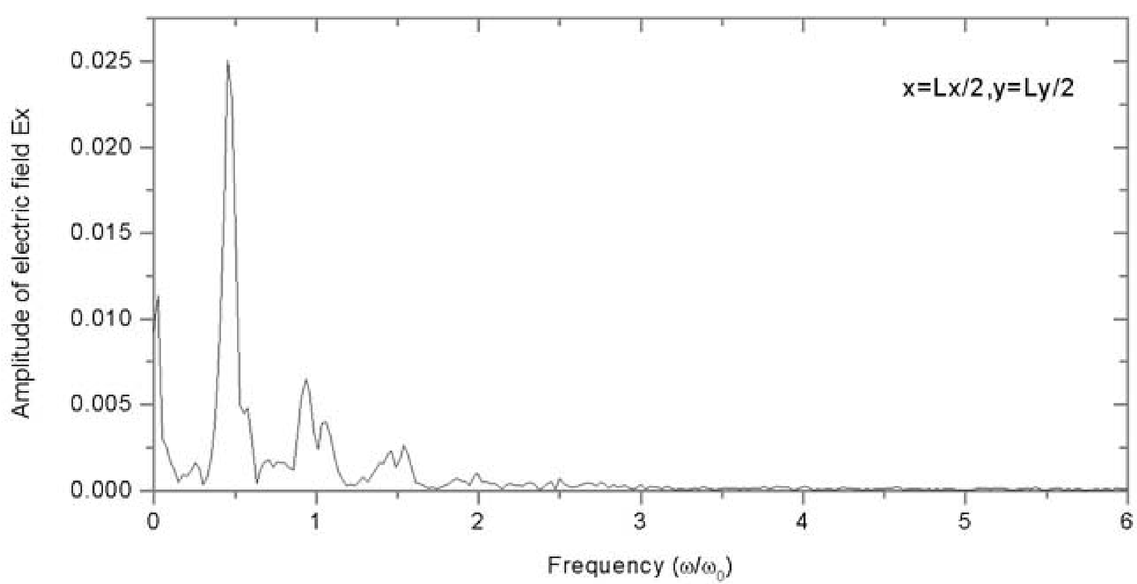


图 4.2.7 垂直入射时离子密度轮廓分布(ωt=206.51)

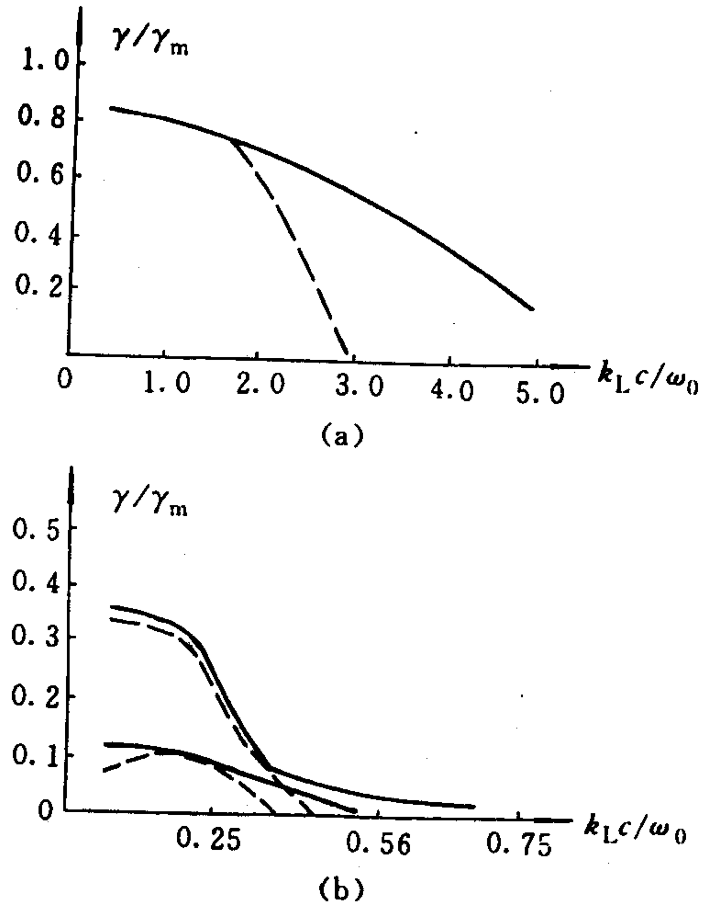
对于**k>>k$_0$**,等离子体波与**k$_0$**和**v$_{os}$**成**45°**角时，其极大增长率

$$\gamma = \frac{kv_{os}}{2}$$



（2）双等离子体衰变的增长率

双等离子体衰变与背景等离子体温度密切相关：温度越高，增长率越小



(b)

图 8.2　均匀等离子体中双等离子体衰变增长率和背景等离子体电子温度 T_e 的关系

(a) T_e = 1keV 时，γ/γ_m 随 $k_L c/\omega_0$ 的变化

(b) T_e = 12keV 和 T_e = 20keV

（2）双等离子体衰变的阈值

- Because of collisional damping and Landau damping, there is a threshold: $\gamma \ge \dfrac{\nu_{ei}}{2}$

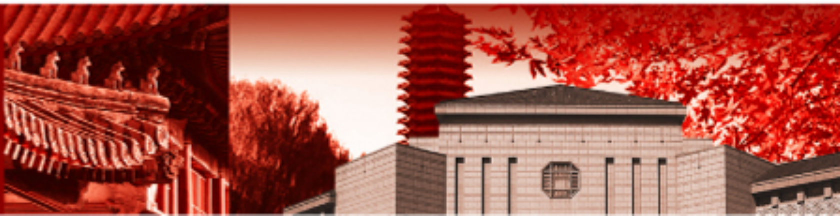
由于碰撞效应对电磁波的能量阻尼：$\gamma_{ei} = \dfrac{\omega_{pe}^{\ 2}}{\omega_0^{\ 2}}\nu_{ei} \approx \dfrac{\nu_{ei}}{2}(n_e \sim n_c/4)$

- Because of plasma density inhomogeneity, there is a threshold:

Rosenbluth criterion $\dfrac{\gamma^2}{|k'v_{g1}v_{g2}|} \ge 1$ ⇒ $(\dfrac{v_{os}}{v_{eth}})^2 \ge \dfrac{12}{k_0 L}$

$B-SRS:(\dfrac{v_{os}}{c})^2 \approx \dfrac{2}{k_0 L}$

$F-SRS:(\dfrac{v_{os}}{c})^2 \approx \dfrac{4\omega_0}{\omega_{pe}k_0 L} \approx \dfrac{8}{k_0 L}(\omega_0 \sim 2\omega_{pe})$



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