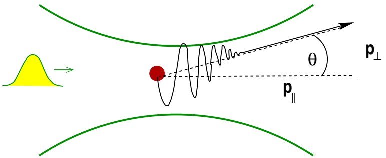
Part IV

Interaction with Single Electrons - Focussed Beams and Thomson Scattering

Ejection of free electrons from focused laser beam



- Ponderomotive force $f_p \propto \nabla_\perp I_L$
- But: $f_p = f_p(r[t], p[t])$ $\rightarrow$ no analytical solution
- Can still determine final energy ΔU

Laser focus: ejection angle (1)

Final kinetic energy of the electron:

$$\Delta U = (\gamma - 1)mc^2. \quad (41)$$

This energy comes from EM field via multiphoton momentum transfer. Parallel momentum is conserved, so:

$$p_{\|} = n\hbar k = \frac{n\hbar\omega}{c} = \frac{\Delta U}{c} = (\gamma - 1)mc. \quad (42)$$

Laser focus: ejection angle (2)

Recall the relationship between $p_{\|}$ and $p_{\perp}$ from Eq. (26) for $\alpha = 1$ (lab frame):

$$p_{\|} = \frac{p_{\perp}^2}{2mc},$$

Emission angle is therefore given by:

$$\tan\theta = \frac{p_{\perp}}{p_{\|}} = \sqrt{\frac{2}{\gamma - 1}}, \qquad (43)$$

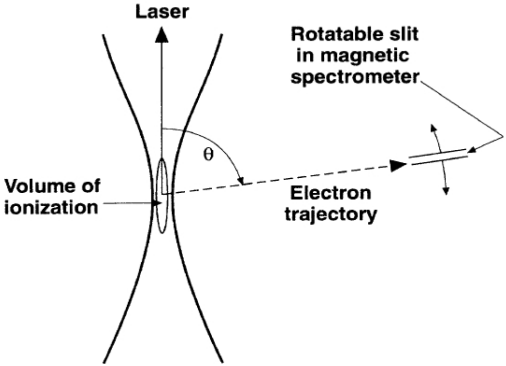
or

$$\cos\theta = \sqrt{\frac{\gamma - 1}{\gamma + 1}}.$$

$\Rightarrow$ one-to-one relationship between exit angle and energy!

Experimental verification of relativistic drift motion

Moore, Meyerhofer *et al.* (1995)



Recap: experimental appearance intensities

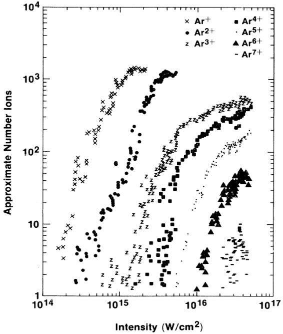


FIG. 1. Approximate number of argon ions detected as a function of peak laser intensity. Similar graphs have been constructed for He, Ne, Kr, and Xe.

$$I_{\mathrm{app}} \simeq 4 \times 10^9 \left(\frac{E_{\mathrm{ion}}}{\mathrm{eV}}\right)^4 Z^{-2} \quad \mathrm{Wcm}^{-2}$$

Appearance intensities from barrier suppression model

- Example: Ne, Z=10, $1s^2 2s^2 2p^6$

Ion	$E_{\rm ion}$ (eV)	$I_{\rm app}$ ($\mathrm{Wcm^{-2}}$)	$2\Phi_{\rm pond}$ (keV)	θ (^{o})
Ne^{3+}	97.1	4.0×10^{16}	7.4	85
Ne^{5+}	157.9	6.7×10^{16}	12.6	83.7
Ne^{7+}	207.3	1.5×10^{17}	47	78

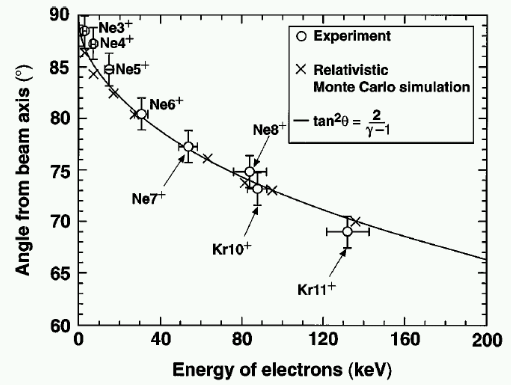
- Ponderomotive potential

$$\Phi_{\rm pond} = \frac{a_0^2 mc^2}{4} = 9.3 \times 10^{-14} I_{\rm app} \lambda_\mu^2$$

- Emission angle from Eq. (43):

$$\theta = tan^{-1} \left(\frac{2mc^2}{E_{\rm kin}/\mathrm{keV}} \right)^{1/2}$$

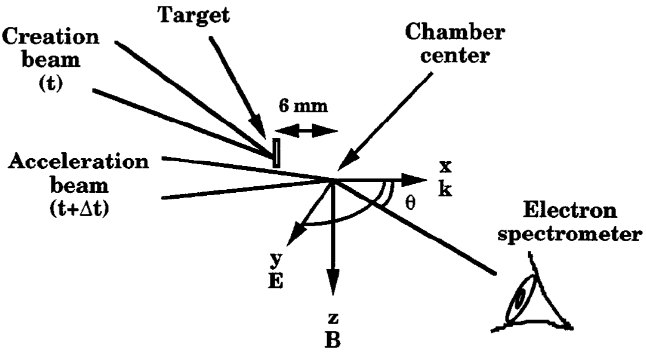
Angle-dependent electron spectra



Ejection from focus in relativistic regime

Malka, Lefebvre & Miquel (1997)

- Limeil experiment (1997):
  $I_L = 0.5 - 1 \times 10^{19}$ Wcm^{-2}, $\lambda = 1$ μm, $\sigma_L = 10$ μm, $\tau_L = 500$fs



- keV electrons drift into main chamber & get accelerated by main pulse

Spectrometer measurements

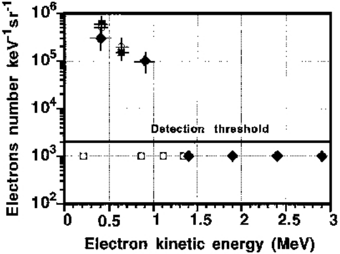


FIG. 2. Electron measurements recorded by the spectrometer at $\theta = 39°$ and $\theta = 46°$ for $a = 3$ and $a = 2$. The laser polarization is horizontal. The maximum energy is $W_{\max} = 0.9 \pm 0.1$ MeV ($a = 3$, $\theta = 39°$, diamonds), 0.63 ± 0.05 MeV ($a = 3$, $\theta = 46°$, circles), and 0.63 ± 0.05 MeV ($a = 2$, $\theta = 46°$, squares).

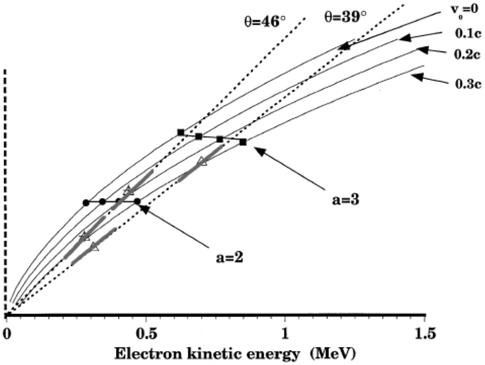
Generalised scattering formula

For electrons with initial velocity $\beta_0 = v_0/c$:

$$\tan\theta = \frac{\sqrt{2(\frac{\gamma}{\gamma_0} - 1)/(1+\beta_0)}}{\gamma - \gamma_0(1-\beta_0)}, \tag{44}$$

where $\gamma_0 = (1-\beta_0^2)^{1/2}$.

Spectrometer ‘simulation’ – polar plot



Malka *et al.*'s analysis

Test-particle model with:

$$
\begin{aligned}
E_y &= \frac{\sigma_0}{\sigma(x)} E_0 \exp\left\{ \frac{-y^2}{\sigma(x)^2} \right\} f(\phi), \\
B_z &= E_y,
\end{aligned}
$$

where $\sigma(x) = \sigma_0(1 + x^2/R_L^2)$, $R_L = \pi\sigma_0^2/\lambda_L$, $\phi = \omega t - kx$

Malka *et al.*'s analysis

Test-particle model with:

$$E_y = \frac{\sigma_0}{\sigma(x)} E_0 \exp\left\{\frac{-y^2}{\sigma(x)^2}\right\} f(\phi),$$
$$B_z = E_y,$$

where $\sigma(x) = \sigma_0(1 + x^2/R_L^2)$, $R_L = \pi\sigma_0^2/\lambda_L$, $\phi = \omega t - kx$
Flawed because fields don't satisfy Maxwell equations!!

$$\nabla \cdot \mathbf{B} \neq 0$$

Corrected fields

Any focused laser pulse described by $A_y = A_0(r)$, must also have

$$B_x = \frac{\partial A_y}{\partial z}.$$

$$\begin{aligned} E_y &= \frac{\sigma_0}{\sigma(x)} E_0 \exp\left\{ \frac{-y^2}{\sigma(x)^2} \right\}, \\ B_z &= E_y, \\ B_x &= \end{aligned}$$

This gives force $v_y B_x$ in the z-direction *of the same order* as f_y

$\Rightarrow$ Get electron *rings* ejected in forward direction MOVIE

The net result is that a symmetric, tightly focused laser will tend to eject rings of electrons in the forward direction.

Laser Acceleration Schemes

Lawson-Woodward (LW) theorem: an isolated, relativistic electron cannot gain energy by interacting with an EM field.
Conditions:

1. the laser field is in vacuum, with no interfering walls or boundaries,
2. the electron is highly relativistic along the acceleration path,
3. no static electric or magnetic fields are present,
4. the interaction region is infinite,
5. ponderomotive forces are neglected.

Proposed vacuum schemes

1. Tightly focused, stationary beam with $I\lambda^2 > 10^{21}$ Wcm$^{-2}\mu$m^2-electron 'capture' (Ho *et al.*, 2000)
2. Tailored laser focus: ponderomotive 'well'
3. Sub-cycle acceleration: 1/2-cycle laser pulse; DC axial field
4. Vacuum beat-wave - optical mixing (Esarey, 1995)
5. Magnetic fields (Katsouleas & Dawson, 1983)

Nonlinear Thomson Scattering

- Accelerated charges act as radiation sources – e.g. synchrotron.
- Similarly, electrons in laser field will re-emit radiation at harmonics of laser frequency.

⇒ Nonlinear, or relativistic Thomson scattering

Classical Thomson Scattering

Power radiated by electron oscillating with velocity in EM field with normalized pump strength a_0:

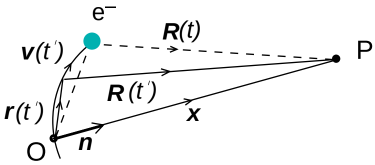
$$P_T = \frac{e^2\omega_0^2 c}{3} a_0^2. \tag{45}$$

Dividing by the Poynting vector for the incoming light, $S = cE_0^2/8\pi$, yields the Thomson scattering cross-section:

$$\sigma_T = \frac{P_T}{S} = \frac{8\pi}{3} r_e^2, \tag{46}$$

where $r_e = e^2/mc^2$ is the classical electron radius.

Relativistic treatment



Liénard-Wiechert potentials for charge in relativistic motion:

$$\phi(\mathbf{x}, t) = \left[\frac{e}{(1-\boldsymbol{\beta}.\mathbf{n})R}\right]_{\rm ret}, \qquad (47)$$

$$\mathbf{A}(\mathbf{x}, t) = \left[\frac{e\boldsymbol{\beta}}{(1-\boldsymbol{\beta}.\mathbf{n})R}\right]_{\rm ret}, \qquad (48)$$

where $\boldsymbol{\beta} = \mathbf{v}/c$, $\mathbf{n}$ is a unit vector from the charge in the direction of observation, and $[\,]_{\rm ret}$ indicates *retarded* time $t' = t - R(t')/c$.

General radiation field

Radiation (electric) field seen at point P due to the charge at $\mathbf{x}(t)$:

$$\mathbf{E}(\mathbf{x},t) = e\left[\frac{\mathbf{n}-\boldsymbol{\beta}}{\gamma^2\kappa R^2}\right]_{\mathrm{ret}} + \frac{e}{c}\left[\frac{\mathbf{n}\times\{(\mathbf{n}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}}}{\kappa^3 R}\right]_{\mathrm{ret}}, \qquad (49)$$

where $\kappa = dt'/dt = 1-\boldsymbol{\beta}.\mathbf{n}$ and $\dot{\boldsymbol{\beta}} = d\boldsymbol{\beta}/dt$ is the acceleration.

Radiated power

Energy flux or Poynting vector at the observation point P:

$$\mathbf{S} = \frac{c}{4\pi}\mathbf{E}\times\mathbf{B} = \frac{c}{4\pi} \mid E \mid^2 \mathbf{n},$$

Radiated power *per unit solid angle*:

$$\frac{dP(t)}{d\Omega} = R^2(\mathbf{S}.\mathbf{n}) = \frac{c}{4\pi} R^2 \mid E \mid^2 . \qquad (50)$$

Intensity distribution

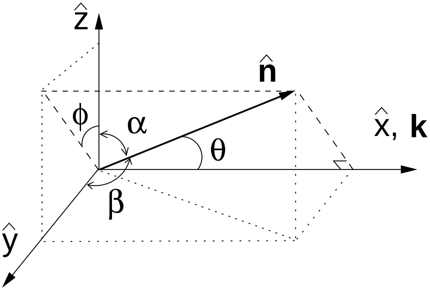
Assume that the field can be expressed as a Fourier integral:

$$\mathbf{E}(t) = \int_{-\infty}^{\infty} e^{-i\omega t}\mathbf{E}(\omega)d\omega.$$

Substituting this into Eq. (49), and applying Parseval's theorem for power spectra to Eq. (50) leads to an *intensity* distribution:

$$\frac{d^2 I}{d\omega d\Omega} = \frac{e^2\omega^2}{4\pi^2 c}\left|\int_{-\infty}^{\infty} \mathbf{n}\times(\mathbf{n}\times\beta)e^{i\omega(t-\mathbf{n.r/c})}dt\right|^2. \qquad (51)$$

Geometry



Special case: periodic motion
Sarachik & Schappert (1970)

Radiated power P per solid angle Ω can be decomposed into multiples m of the fundamental frequency ω_0, whereby:

$$\begin{aligned} \frac{dP_m}{d\Omega} &= \frac{\omega_0^2}{2\pi}\frac{d^2 I_m}{d\omega d\Omega} \\ &= \frac{e^2\omega_0^4 m^2}{(2\pi c)^3}\left|\int_0^{2\pi/\omega_0} \mathbf{n}\times(\mathbf{n}\times\mathbf{v})e^{im\omega_0(t-\mathbf{n.r}/\mathbf{c})}dt\right|^2. \quad (52) \end{aligned}$$

Radiation from relativistic quiver motion

Apply formula directly to the periodic particle orbits Eq. (35) in the *average rest frame* to get power spectrum:

$$\frac{dP_R^m}{d\Omega_R} = \frac{2m^2 A(\omega_R^2)}{\gamma_0^2}\left[\frac{\cot^2\theta_R}{2q^2} J_m^2(\sqrt{2}q\, m\sin\theta) + {J'}_m^2(\sqrt{2}q\, m\sin\theta_R)\right], \tag{53}$$

where

$$A(\omega_R^2) = \frac{e^2\omega_R^2 a_0^2}{8\pi c},$$

J_m is the usual Bessel function and J'_m its derivative; θ_R is the angle between the $\mathbf{n}$ and the laser wave-vector in the average rest frame; $q = a_0/\gamma_0$, $\omega_R = \omega_0/\gamma_0$.

Observed radiation spectrum in lab frame

Lorentz-transform result back to the lab frame to get the *observed* power spectrum.

$$\frac{dP_L^m}{d\Omega_L} = D(a_0, \theta_L)\frac{dP_R^m}{d\Omega_R}, \tag{54}$$

where

$$D(a_0, \theta_L) = \frac{\gamma_0^4}{\left(1 + \frac{a_0^2}{2}\sin^2\frac{\theta_L}{2}\right)^4}.$$

The spectrum also no longer consists of integer harmonics of ω_0, but at *shifted* frequencies given by:

$$\omega_L^m = \frac{m\omega_0}{\left(1 + \frac{a_0^2}{2}\sin^2\frac{\theta_L}{2}\right)}. \tag{55}$$

Strongly relativistic limit: C-polarized light

Extreme intensity limit: ($a_0 \gg 1$), radiation is predominantly forward and confined to an angle

$$\theta_L = \sqrt{8}/a_0$$

from the axis of propagation. *Headlamping* effect: $\theta_L = \theta_p$ orbit pitch-angle for C-light.
Also get harmonic cutoff at:

$$M = m_{\rm max} = 3(a_0^2/2)^{3/2}.$$

Linearly polarized light

Expand the angular radiation pattern Eq. (52) for $a_0 < 1$. The total scattered power in the first three harmonics — integrated over solid angle in the laboratory frame — has the following leading terms:

$$\begin{aligned} P_1 &\simeq W_0 \frac{8\pi}{3} a_0^2, \\ P_2 &\simeq W_0 \frac{14\pi}{5} a_0^4, \\ P_3 &\simeq W_0 \frac{621\pi}{224} a_0^6, \end{aligned} \tag{56}$$

where $W_0 = e^2\omega_0^2 c/8\pi$ is a characteristic scattered power per electron for a given laser frequency ω_0.

Seeing figures-of-eight

Chen, Umstadter *et al.*(Nature, 1998)

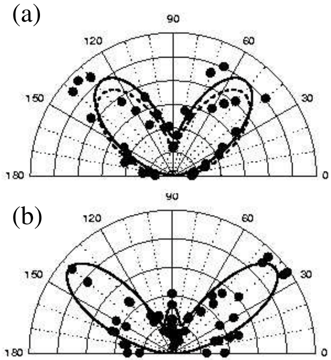


Figure: Angular measurements (points) of a) 2nd harmonic light and b) 3rd harmonic light generated by relativistic electrons in a high intensity laser focus.

Nonlinear Compton scattering

Consider recoil energy and momentum acquired by the electron.

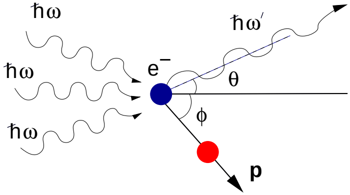


Figure: Geometry for multiphoton-electron scattering event in which electron is initially at rest.

- Classical Thomson scattering: $\hbar\omega/mc^2 \ll 1$ – ignore recoil effects.
- Compton scattering: $\hbar\omega/mc^2 \sim 1 \Rightarrow$ gain in momentum for the electron; loss of photon energy

Single-photon scattering

For single-photon scattering, the energy change ΔE is easily found by momentum and energy conservation:

$$\Delta E = \frac{\frac{\hbar\omega}{mc^2}(1-\cos\theta)}{1+\frac{\hbar\omega}{mc^2}(1-\cos\theta)}\hbar\omega, \tag{57}$$

which corresponds to the usual wavelength shift of the scattered photon,

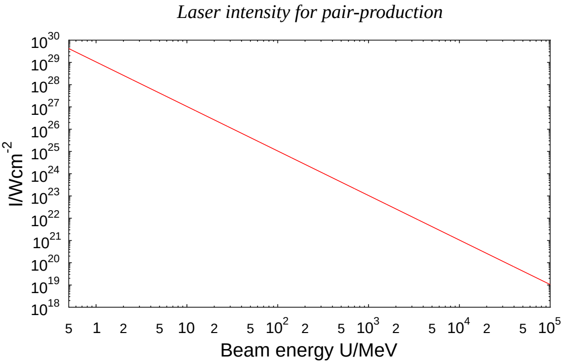
$$\Delta\lambda = \lambda' - \lambda = \lambda_c(1-\cos\theta),$$

where $\lambda_c = h/mc = 0.0243\text{Å}$ is the Compton wavelength.

Multiphoton scattering

For relativistic electrons, the photon frequency gets upshifted by γ in the electron rest frame.
Relevant parameter:

$$\gamma n\hbar\omega/mc^2.$$



Observation of multiphoton Compton scattering

Bula *et al.*, 1996

- Experiment at the Stanford Linear Accelerator Center (SLAC): where a 47 GeV ($\gamma \simeq 92000$) 'collided' with a TW, 1 ps, 1 μm laser pulse.
- Nd:glass laser photons have $\hbar\omega \simeq 1$ eV, giving $\gamma\hbar\omega/mc^2 \sim 1$.

→ Significant multiple scattering at 10^{18} Wcm^{-2}, which scales as $P_n \sim a_0^{2n}$.

- Broad agreement with QED theory.

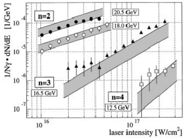


FIG. 5. The normalized yield of scattered electrons of energies corresponding to $n = 2$, 3, and 4 infrared laser photons per interaction versus the intensity of the laser field at the interaction point. The bands represent a simulation of the experiment, including 30% uncertainty in laser intensity and 10% uncertainty in N_γ.