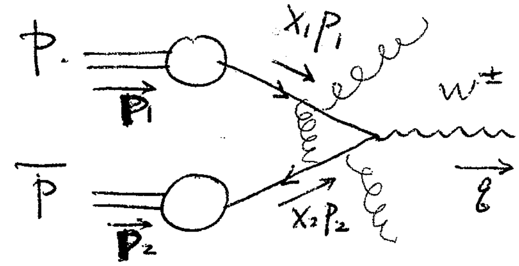
$\boxed{P\bar{P} \to W^{\pm}}$



$(q^2 = M^2)$

(1) Total cross section (HADRON LEVEL)

$$\sigma(P\bar{P}\to W) = \sum_{i,j}\int_{\tau_0}^{1} dx_1 \int_{\frac{\tau_0}{x_1}}^{1} dx_2 \left[q_{i/P}(x_1)\,\bar{q}_{j/\bar{P}}(x_2) + \bar{q}_{j/P}(x_1)\,q_{i/\bar{P}}(x_2)\right] \hat{\sigma}(q_i q_j \to W)$$

Where

$$\tau_0 = \frac{M^2}{S}, \quad (\sqrt{S} \text{ — the c.m. energy of } P\bar{P})$$

$$\hat{s} = (p_1+p_2)^2 = 2p_1\cdot p_2 = x_1 x_2 \cdot 2P_1\cdot P_2 = x_1 x_2 \cdot S$$

$$\hat{\tau} = \frac{M^2}{\hat{s}} = \frac{M^2}{x_1 x_2 S} = \frac{\tau_0}{x_1 x_2}$$

(2) Subprocess cross section($\hat{\sigma}$)

(i) $\underline{O(\alpha_s)}$ corrections

$$\hat{\sigma}^{NLO}_{Virt} = \hat{\sigma}_0 \cdot \frac{\alpha_s}{2\pi}\,\delta(1-\hat{\tau})\left(\frac{4\pi\mu^2}{M^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left\{-\frac{2}{\epsilon^2} - \frac{3}{\epsilon} - 7 - \frac{\pi^2}{3} + \delta_{scheme}\right\}$$

where $\hat{\sigma}_0$ — Tree level cross section, $\alpha_s = \frac{g_3^2}{4\pi}$

$$\delta_{scheme} = \begin{cases} 0, & \text{DRED} \\ -1 & \text{Naive } \gamma_5,\ \text{HVBM} \end{cases}$$

$$\hat{\sigma}^{NLO}_{REAL} = \hat{\sigma}_0 \cdot \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{M^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left\{\frac{2}{\epsilon^2}\delta(1-\hat{\tau}) - \frac{2}{\epsilon}\frac{1+\hat{\tau}^2}{(1-\hat{\tau})_+} + 4(1+\hat{\tau}^2)\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ + 4(1-\hat{\tau})\right\}$$

$$\Rightarrow \hat{\sigma}^{NLO}_{Virt} + \hat{\sigma}^{NLO}_{REAL} \equiv \hat{\sigma}^{(1)}$$

$$= \hat{\sigma}_0 \frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu^2}{M^2}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left\{-\frac{2}{\epsilon}\left(\frac{1+\hat{\tau}^2}{1-\hat{\tau}}\right)_+ + 4(1-\hat{\tau}) + 4(1+\hat{\tau}^2)\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ + \left(-7-\frac{\pi^2}{3}+\delta_{scheme}\right)\delta(1-\hat{\tau})\right\}$$

where we use

$$-\frac{2}{\epsilon}\left[\frac{1+\hat{\tau}^2}{(1-\hat{\tau})_+}+\frac{3}{2}\delta(1-\hat{\tau})\right]=-\frac{2}{\epsilon}\left(\frac{1+\hat{\tau}^2}{1-\hat{\tau}}\right)_+$$

(3) Redefine the Parton Distribution Function (PDF)

PDF's evalution equation in scale Q is

$$\frac{d\,q(x,Q^2)}{d\ln Q^2}=\frac{\alpha_s}{2\pi}\int\frac{d\xi}{\xi}\underbrace{\left(\frac{1+\left(\frac{x}{\xi}\right)^2}{1-\left(\frac{x}{\xi}\right)}\right)_+}_{\text{splitting kernel}}C_F\cdot q(\xi,Q^2)$$

The relation between the scale dependent and bare PDF is

$$f_i^A(x,Q_{PDF}^2)=\int_x^1\frac{dz}{z}\left[\delta_{ij}\,\delta(1-z)+\frac{\alpha_s}{2\pi}R_{i\leftarrow j}(z,Q_{PDF}^2)\right]f_{j,\text{bare}}^A\left(\frac{x}{z}\right)$$

where

$$R_{i\leftarrow j}(z,Q_{PDF}^2)=-\frac{1}{\epsilon}P_{i\leftarrow j}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left(\frac{4\pi\mu^2}{Q_{PDF}^2}\right)^{\epsilon}\qquad(\overline{MS}\text{ scheme})$$

DGLAP

$P_{i\leftarrow j}$ is the Altarelli-Parisi splitting function.
Q_{PDF} is the scale that the PDF is to be evaluated.

Thus,

$$(*)\quad\boxed{f_{i,\text{bare}}^A(x)=f_i^A(x,Q_{PDF}^2)-\frac{\alpha_s}{2\pi}\int_x^1\frac{dz}{z}R_{i\leftarrow j}(z,Q_{PDF}^2)f_{j,\text{bare}}^A\left(\frac{x}{z}\right)}$$

Then, we define PDF in the expansion of α_s,

$$f_{i/a,\text{bare}}=\underbrace{f_{i/a,\text{bare}}^{(0)}}_{\alpha_s^{(0)}}+\underbrace{f_{i/a,\text{bare}}^{(1)}}_{\alpha_s^{(1)}}$$

So in the $\overline{MS}$ scheme,

$$f^{(0)}_{i/a,\,\text{bare}} = f^{(0)}_{i/a,\,\text{ren}}$$

$$f^{(1)}_{i/a,\,\text{bare}} = -\frac{\alpha_s}{2\pi} R^{(1)}_{i\leftarrow j} \otimes f_{j/a,\,\text{ren}}.$$

← "convolution notation"

$$C(z) = A \otimes B = \int_z^1 \frac{dy}{y} A\left(\frac{z}{y}\right) B(y) = \int_z^1 \frac{dy}{y} A(y) B\left(\frac{z}{y}\right)$$

Then,

$$\sigma(p\bar{p} \to W) = \sum_n \left(\frac{\alpha_s}{2\pi}\right)^n \sigma^{(n)}_{p\bar{p}}$$

$$= q_{i/p,\,\text{bare}} \otimes \hat{\sigma}_{ij} \otimes \bar{q}_{j/\bar{p},\,\text{bare}}$$

$$= q^{(0)}_{i/p,\,\text{bare}} \otimes \hat{\sigma}^{(0)}_{ij} \otimes \bar{q}^{(0)}_{j/\bar{p},\,\text{bare}} \quad \leftarrow \text{Tree level}$$

$$\left.\begin{aligned} &+ q^{(1)}_{i/p,\,\text{bare}} \otimes \hat{\sigma}^{(0)}_{ij} \otimes \bar{q}^{(0)}_{j/\bar{p},\,\text{bare}} \\ &+ q^{(0)}_{i/p,\,\text{bare}} \otimes \hat{\sigma}^{(1)}_{ij} \otimes \bar{q}^{(0)}_{j/\bar{p},\,\text{bare}} \\ &+ q^{(0)}_{i/p,\,\text{bare}} \otimes \hat{\sigma}^{(0)}_{ij} \otimes \bar{q}^{(1)}_{j/\bar{p},\,\text{bare}} \end{aligned}\right\} \text{NLO}$$

$$= q^{(0)}_{i/p,\,\text{ren}} \otimes \hat{\sigma}^{(0)}_{ij} \otimes \bar{q}^{(0)}_{j/\bar{p},\,\text{ren}}$$

$$A \left\{\begin{aligned} &+ \left(-\frac{\alpha_s}{2\pi} R^{(1)}_{i\leftarrow i'} \otimes q^{(0)}_{i'/p,\,\text{ren}} \otimes \hat{\sigma}^{(0)}_{ij} \otimes \bar{q}^{(0)}_{j/\bar{p},\,\text{ren}}\right) \\ &+ \left(-\frac{\alpha_s}{2\pi} q^{(0)}_{i/p,\,\text{ren}} \otimes \hat{\sigma}^{(0)}_{ij} \otimes R^{(1)}_{j\leftarrow j'} \otimes \bar{q}^{(0)}_{j'/\bar{p},\,\text{ren}}\right) \end{aligned}\right.$$

$$B \left\{ + q^{(0)}_{i/p,\,\text{ren}} \otimes \hat{\sigma}^{(1)}_{ij} \otimes \bar{q}^{(0)}_{j/\bar{p},\,\text{ren}} \right.$$

(4) Consider

$$\sigma(p\bar{p}\to W^\pm) = \int dx_1 \int dx_2 \left[q_{i/p}(x_1)\, \bar{q}_{j/\bar{p}}(x_2) \right] \hat{\sigma}(q_i \bar{q}_j \to W^\pm)$$

$$\hat{\sigma}_{ij}^{(0)} = \hat{\sigma}_0\, \delta(1-\hat{\tau})$$

$$\hat{\sigma}_{ij}^{(1)} = \hat{\sigma}_0 \left(\frac{\alpha_s}{2\pi} C_F\right) \Big\{ -\frac{2}{\epsilon}\cdot\frac{1}{C_F} P^{(1)}_{q\leftarrow q}(\hat{\tau}) \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{Q^2_{PDF}}\right)\left(\frac{Q^2_{PDF}}{M^2}\right)^{\epsilon} + 4(1-\tau) + 4(1+\hat{\tau}^2)\left(\frac{\log(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ + \left(-7-\frac{\pi^2}{3}+\delta_{scheme}\right)\delta(1-\hat{\tau}) \Big\}$$

$$= -\frac{2}{\epsilon}\hat{\sigma}_0 \cdot \frac{\alpha_s}{2\pi} P^{(1)}_{q\leftarrow q}(\hat{\tau}) \frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)} \left(\frac{4\pi\mu^2}{Q^2_{PDF}}\right)^{\epsilon}$$

$$+ \hat{\sigma}_0\left(\frac{\alpha_s}{2\pi} C_F\right) \Big\{ 2\ln\left(\frac{Q^2_{PDF}}{M^2}\right)^{\epsilon} \left(\frac{1+\hat{\tau}^2}{1-\hat{\tau}}\right)_+ + 4(1-\hat{\tau}) + 4(1+\hat{\tau}^2)\left(\frac{\log(1-\hat{\tau})}{1-\hat{\tau}}\right)_+$$

$$+ \left(-7-\frac{\pi^2}{3}+\delta_{scheme}\right)\delta(1-\hat{\tau}) \Big\}$$

$$= 2\hat{\sigma}_0 \frac{\alpha_s}{2\pi} R^{(1)}_{q\leftarrow q}(\hat{\tau}, Q^2_{PDF}) + \hat{\sigma}^{(1)}_{finite}$$

(I) $$B = q^{(0)}_{i/p,ren} \otimes \hat{\sigma}^{(1)}_{ij} \otimes \bar{q}^{(0)}_{j/\bar{p},ren}$$

$$= \underbrace{2\cdot\frac{\alpha_s}{2\pi}\, q^{(0)}_{i/p,ren} \otimes \hat{\sigma}_0 \otimes \bar{q}^{(0)}_{j/\bar{p},ren}\, R^{(1)}_{q\leftarrow q}(\hat{\tau}, Q^2_{PDF})}_{\text{singular}} + \underbrace{q^{(0)}_{i/p,ren} \otimes \hat{\sigma}^{(1)}_{finite} \otimes \bar{q}^{(0)}_{j/\bar{p},ren}}_{\text{finite}}$$

(II) $$A = -\frac{\alpha_s}{2\pi} R^{(1)}_{i\leftarrow i'} \otimes q^{(0)}_{i/p,ren} \otimes \hat{\sigma}^{(0)}_{ij} \otimes \bar{q}^{(0)}_{j/\bar{p},ren} - \frac{\alpha_s}{2\pi}\, q^{(0)}_{i/p,ren} \otimes \hat{\sigma}^{(0)}_{ij} \otimes R^{(1)}_{j\leftarrow j'} \otimes \bar{q}^{(0)}_{j'/\bar{p},ren}$$

$$= -2\cdot\frac{\alpha_s}{2\pi}\, q^{(0)}_{i/p,ren} \otimes \hat{\sigma}_0 \otimes \bar{q}_{j/\bar{p}} \cdot R^{(1)}_{q\leftarrow q}(\hat{\tau}, Q^2_{PDF})$$

Hence, we could see that $A+B = q^{(0)}_{i/p,ren} \otimes \hat{\sigma}^{(1)}_{finite} \otimes \bar{q}^{(0)}_{j/\bar{p},ren}$ and it is fini

This means the $\frac{1}{\epsilon}$ poles cancel.

(5) Therefore, up to Next-to-leading-order, the total cross section should be.

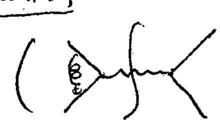
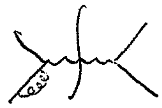
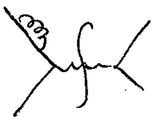
$$\sigma(p\bar{p}\to W^{\pm}) = \sum_{i,j}\int_{\tau_0}^{1} dx_1 \int_{\frac{\tau_0}{x_1}}^{1} dx_2 \left[q^{ren}_{i/p}(x_1,Q^2)\, \bar{q}^{ren}_{j/\bar{p}}(x_2,Q^2) + \bar{q}^{ren}_{j/\bar{p}}(x_1,Q^2)\, q^{ren}_{i/\bar{p}}(x_2,Q^2)\right] \times \hat{\sigma}_0$$

$$\times \Big\{ \delta(1-\hat{\tau}) \qquad \text{Tree level}$$

$$\underbrace{+\frac{\alpha_s}{2\pi} C_F \left[2\ln\left(\frac{M^2}{Q^2}\right)\left(\frac{1+\hat{\tau}^2}{1-\hat{\tau}}\right)_+ + 4(1-\hat{\tau}) + 4(1+\hat{\tau}^2)\left(\frac{\log(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ + \left(-7-\frac{\pi^2}{3}+\delta_{scheme}\right)\delta(1-\hat{\tau})\right.}_{\text{NLO}}$$

For Myself

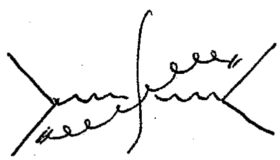
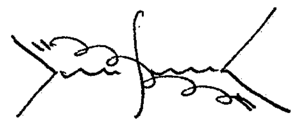
1. $\boxed{\text{Virtual Corrections}}$

$2\,($  $+\frac{1}{2}$  $+\frac{1}{2}$  $)$

$$2\tilde{\sigma}_0\,\frac{1}{16\pi^2}\,\delta\left(1-\frac{M^2}{\hat{s}}\right)\times\left(\frac{4\pi\mu^2}{M^2}\right)^{\epsilon_{IR}}\frac{\Gamma(1-\epsilon_{IR})}{\Gamma(1-2\epsilon_{IR})}\left\{-\frac{2}{\epsilon_{IR}^2}-\frac{3}{\epsilon_{IR}}-7-\frac{\pi^2}{3}+\delta_{scheme}\right\}$$

where $\delta_{scheme}=\begin{cases}0, & \text{DRED}\\ -1, & \text{Naive } \gamma_5,\ \text{HVBM}\end{cases}$

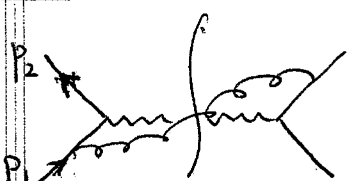
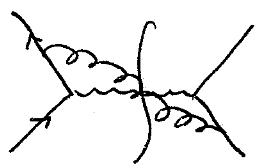
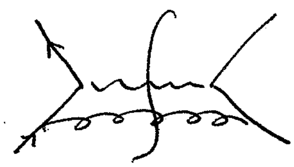
2. $\boxed{\text{Soft - Limit}}$

 $+$ 

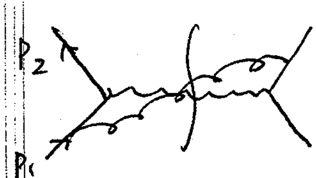
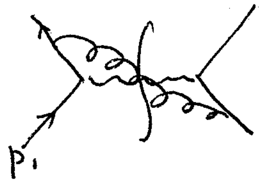
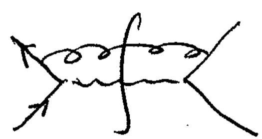
$$2\tilde{\sigma}_0\,\frac{1}{16\pi^2}\left(\frac{4\pi\mu^2}{\hat{s}}\right)^{\epsilon_{IR}}\frac{\Gamma(1-\epsilon_{IR})}{\Gamma(1-2\epsilon_{IR})}\left\{\frac{2}{\epsilon_{IR}^2}\delta(1-\hat{\tau})-\frac{4}{\epsilon_{IR}}\frac{1}{(1-\hat{\tau})_+}+8\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+\right\}$$

3. $\boxed{\text{Collinear-Limit}}$

(1) $P_3 /\!/ P_1$

 $+$  $+$ 

(2) $P_3 /\!/ P_2$

 $+$  $+$ 

mixed-diagrams' contributions:

$$2\tilde{\sigma}_0\left(\frac{4\pi\mu^2}{\hat{s}}\right)^{\epsilon}\cdot\frac{1}{16\pi^2}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\cdot\left\{\frac{4}{\epsilon^2}\delta(1-\hat{\tau})+\frac{4}{\epsilon}\frac{1+\hat{\tau}}{(1-\hat{\tau})_+}+8(1+\hat{\tau})\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+\right\}$$

unmixed-diagram's contributions:

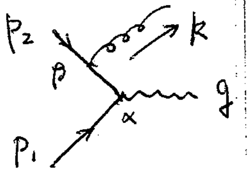
$$2\tilde{\sigma}_0\left(\frac{4\pi\mu^2}{\hat{s}}\right)^{\epsilon}\cdot\frac{1}{8\pi^2}\cdot\frac{1}{\Gamma(1-\epsilon)}\cdot(1-\hat{\tau})^{1-2\epsilon}\,B(-\epsilon,2-\epsilon)\,\underline{(1-\epsilon)}$$

The last item "$1-\epsilon$" comes from the Dirac Matrices Algebra in n-dimension

Use $(1-x)^{-1-2\epsilon}=-\frac{1}{2\epsilon}\delta(1-x)+\frac{1}{(1-x)_+}-2\epsilon\left(\frac{\ln(1-x)}{1-x}\right)_+$, then

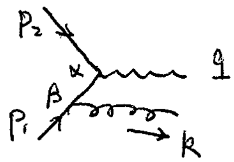
$$\Rightarrow 2\tilde{\sigma}_0\left(\frac{4\pi\mu^2}{\hat{s}}\right)^{\epsilon}\frac{1}{16\pi^2}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left\{-\frac{2}{\epsilon}(1-\hat{\tau})-4(1-\hat{\tau})^2\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ +2(1-\hat{\tau})\right\}$$

4. $\boxed{\text{THE FULL REAL CORRECTIONS}}$



$$iM_1 = \bar{v}(p_2,h_2)\left[(-ig_s\gamma^\beta)\frac{-i(\not p_2-\not k)}{(p_2-k)^2+i\varepsilon}\, i\Gamma^\alpha\right]\bar{u}(p_1,h_1)\, T^a_{ij}\, \epsilon^*_\alpha(q)\,\epsilon^*_\beta(k)$$

$$M_1 = -\bar{v}(p_2,h_2)\left[\gamma^\beta(\not p_2-\not k)\Gamma^\alpha\right]\bar{u}(p_1,h_1)\cdot\frac{g_s T^a_{ij}}{(p_2-k)^2}\,\epsilon^*_\alpha(q)\,\epsilon^*_\beta(k)$$



$$iM_2 = \bar{v}(p_2,h_2)\left[i\Gamma^\alpha\frac{i(\not p_1-\not k)}{(p_1-k)^2+i\varepsilon}(-ig_s T^a_{ij}\gamma^\beta)\right]\bar{u}(p_1,h_1)\,\epsilon^*_\alpha(\vec{q})\,\epsilon^*_\beta(\vec{k})$$

$$M_2 = \bar{v}(p_2,h_2)\left[\Gamma^\alpha(\not p_1-\not k)\gamma^\beta\right]\bar{u}(p_1,h_1)\,\frac{g_s T^a_{ij}}{(p_1-k)^2}\,\epsilon^*_\alpha(\vec{q})\,\epsilon^*_\beta(\vec{k})$$

$$\Gamma^\mu = \frac{g_w}{2\sqrt{2}}\cdot\gamma^\mu(1-\gamma^5)\,,\quad g_w = \frac{e}{s_w}$$

Thus, squaring and summing over spins and colors, we obtain

$$\sum|M_R|^2 = M_{11} + M_{22} + M_{33}$$

$$M_{11} = |M_{1R}|^2_\Sigma = \mathrm{Tr}\left[\not p_2\gamma^\beta(\not p_2-\not k)\gamma^\alpha\not p_1\gamma_\alpha(\not p_2-\not k)\gamma_\beta\right]\times\frac{1}{(2p_2\cdot k)^2}$$

$$M_{22} = |M_{2R}|^2_\Sigma = \mathrm{Tr}\left[\not p_2\gamma^\alpha(\not p_1-\not k)\gamma^\beta\not p_1\gamma_\beta(\not p_1-\not k)\gamma_\alpha\right]\times\frac{1}{(2p_1\cdot k)^2}$$

$$M_{12} = |2M_1M^*_{2R}|^2_\Sigma = -2\,\mathrm{Tr}\left[\not p_2\gamma^\beta(\not p_2-\not k)\gamma^\alpha\not p_1\gamma_\beta(\not p_1-\not k)\gamma_\alpha\right]\times\frac{1}{2p_1\cdot k\; 2p_2\cdot k}$$

Here we omit the coupling const and color factor.

In the $q\bar{q}$ c.m. frame, we ~~st~~ choose the particles moving along z-axis, and the Mandelstam variables are

$$\hat{s} = (p_1+p_2)^2 = 2p_1\cdot p_2$$
$$\hat{t} = (p_1-p_3)^2 = -2p_1\cdot p_3$$
$$\hat{u} = (p_2-p_3)^2 = -2p_2\cdot p_3$$

We should do the γ-matrices in the $N=4-2\epsilon$ dimensions,

$$\{\gamma^\mu,\gamma^\nu\} = 2g^{\mu\nu}\mathbb{1}_N$$

By using relations

$$\gamma^\mu\not a\gamma_\mu = -2(1-\epsilon)\not a$$
$$\gamma^\mu\not a\not b\gamma_\mu = 4a\cdot b - 2\epsilon\not a\not b$$
$$\gamma^\mu\not a\not b\not c\gamma_\mu = -2\not c\not b\not a + 2\epsilon\not a\not b\not c$$
$$\mathrm{Tr}(\not a\not b\not c\not d) = 4(a\cdot b\, c\cdot d + a\cdot d\, b\cdot c - a\cdot c\, b\cdot d)$$

We obtain the traces as following.

$$M_{11} = 16(1-\epsilon)^2 \frac{\hat{t}}{\hat{u}}$$

$$M_{22} = 16(1-\epsilon)^2 \cdot \frac{\hat{u}}{\hat{t}}$$

$$M_{12} = -32(1-\epsilon)[-\hat{s}M^2 + \epsilon\,\hat{t}\hat{u}]/\hat{t}\hat{u}$$

Thus.

$$\sum |M_k|^2 = 16(1-\epsilon)\left[(1-\epsilon)\left(\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}}\right) + 2\cdot\frac{\hat{s}M^2}{\hat{t}\hat{u}} - 2\epsilon\right]$$

The coupling constants and color factors are

$$\mu^{2\epsilon}\cdot\left(\frac{g_W}{2\sqrt{2}}\right)^2\cdot g_s^2\,.\,\mathrm{Tr}(T^AT^A) = \mu^{2\epsilon}\cdot\frac{g_W^2}{8}\cdot g_s^2\cdot\mathrm{Tr}(T^AT^A)$$

$$\mathrm{Tr}(T^AT^A) = C_A C_F = 4\,.$$

The spin and color average factor are

$$\left(\frac{1}{2}\cdot\frac{1}{2}\right) \quad \text{and} \quad \left(\frac{1}{3}\cdot\frac{1}{3}\right) \,.$$

Hence.

$$\overline{|M|^2} = \left(\frac{1}{2}\cdot\frac{1}{2}\right)\times\left(\frac{1}{3}\cdot\frac{1}{3}\,C_A C_F\right)\cdot\mu^{2\epsilon}\frac{g_W^2}{8}g_s^2\cdot 16(1-\epsilon)\left[(1-\epsilon)\left(\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}}\right) + 2\frac{\hat{s}M^2}{\hat{t}\hat{u}} - 2\epsilon\right]$$

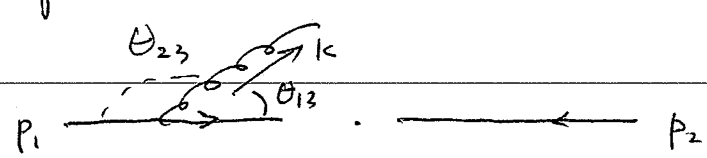
$$= \frac{1}{9}\cdot\mu^{2\epsilon}\,2\,g_W^2 g_s^2\cdot(1-\epsilon)\left[(1-\epsilon)\left(\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}}\right) + 2\frac{\hat{s}M^2}{\hat{t}\hat{u}} - 2\epsilon\right]$$

And the phase-space integration

$$\int d^N p\, \delta^+(p^2) = \frac{2^{1-2\epsilon}\pi^{1-\epsilon}}{\Gamma(1-\epsilon)} \int_0^\infty d|p|\cdot|p|^{1-2\epsilon}\int_0^1 dy\,[y(1-y)]^{-\epsilon}$$

<u>Kinematics.</u>

In the CM frame of $q\bar{q}$, Let P_1 and P_2 propagate along Z-axis. and gluon move along the $(N-1)^{th}$ direction. then $k = (|k|, \cdots |k|\cos\theta)$



Thus.

$$\hat{s} = 2P_1\cdot P_2$$

$$\hat{t} = -2P_1\cdot P_3 = -\hat{s}\left(1-\frac{M^2}{\hat{s}}\right)(1-y)$$

$$\hat{u} = -2P_2\cdot P_3 = -\hat{s}\left(1-\frac{M^2}{\hat{s}}\right) y$$

where $y = \frac{1}{2}(1+\cos\theta)$

Thus,

$$(1-\epsilon)\left(\frac{\hat{t}}{\hat{u}}+\frac{\hat{u}}{\hat{t}}\right)+2\frac{\hat{s}M^2}{\hat{t}\hat{u}}-2\epsilon$$

$$=(1-\epsilon)\left(\frac{1-y}{y}+\frac{y}{1-y}\right)+\frac{2\tau}{(1-\tau)^2}\cdot\frac{1}{y(1-y)}-2\epsilon \qquad \left(\hat{\tau}=\frac{M^2}{\hat{s}}\right)$$

So,

$$\sigma^{NLO}=\overline{|M|^2}(p.s)$$

$$=\mu^{2\epsilon}\cdot\int\frac{d^nk}{(2\pi)^n}\cdot(2\pi)\delta^+(k^2)\cdot\overline{|M|^2}\,\delta(\hat{s}-M^2-2\sqrt{\hat{s}}\,|\mathbf{K}|)$$

$$=\mu^{2\epsilon}\cdot\frac{2^{1-2\epsilon}\pi^{1-\epsilon}}{(2\pi)^{n-1}\Gamma(1-\epsilon)}\int_0^\infty d|k|\cdot|k|^{1-2\epsilon}\cdot\delta(\hat{s}-M^2-2\sqrt{\hat{s}}\,|k|)\cdot\left[\frac{1}{9}\,2g_W^2g_s^2(1-\epsilon)\right]$$

$$\int_0^1 dy\,[y(1-y)]^{-\epsilon}\cdot\left[(1-\epsilon)\left(\frac{1-y}{y}+\frac{y}{1-y}\right)+\frac{2\hat{\tau}}{(1-\hat{\tau})^2}\cdot\frac{1}{y(1-y)}-2\epsilon\right]$$

$$=\frac{1}{16\pi^2}\left(\frac{4\pi\mu^2}{s}\right)^\epsilon\cdot\frac{1}{\Gamma(1-\epsilon)}(1-\hat{\tau})^{1-2\epsilon}\cdot\left[\frac{1}{9}\cdot 2g_W^2g_s^2(1-\epsilon)\right]$$

$$\cdot\left\{\int_0^1 dy\cdot y^{-\epsilon}(1-y)^{-\epsilon}\left[(1-\epsilon)\left(\frac{1-y}{y}+\frac{y}{1-y}\right)+\frac{2\hat{\tau}}{(1-\hat{\tau})^2}\frac{1}{y(1-y)}-2\epsilon\right]\right\}$$

Use $\displaystyle\int_0^1 dy\; y^\alpha(1-y)^\beta=\frac{\Gamma(1+\alpha)\Gamma(1+\beta)}{\Gamma(2+\alpha+\beta)}$,

We obtain the above y integral as

$$\{\;\}=-\frac{2}{\epsilon}\frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}\left[(1-\epsilon)^2+\frac{2\hat{\tau}}{(1-\hat{\tau})^2}\right]$$

Use $\displaystyle (1-x)^{-1-2\epsilon}=-\frac{1}{2\epsilon}\delta(1-x)+\frac{1}{(1-x)_+}-2\epsilon\left(\frac{\ln(1-x)}{1-x}\right)_+ +o(\epsilon^2)$.

$$(1-\hat{\tau})^{1-2\epsilon}=(1-\hat{\tau})^2(1-\hat{\tau})^{-1-2\epsilon}$$

$$=(1-\hat{\tau})^2\left[-\frac{1}{2\epsilon}\delta(1-\hat{\tau})+\frac{1}{(1-\hat{\tau})_+}-2\epsilon\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+\right]$$

$$=\frac{(1-\hat{\tau})^2}{(1-\hat{\tau})_+}-(1-\hat{\tau})^2\cdot 2\epsilon\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+$$

$$-\frac{2}{\epsilon}(1-\epsilon)^2(1-\hat{\tau})^{1-2\epsilon}=-\frac{2}{\epsilon}\frac{(1-\hat{\tau})^2}{(1-\hat{\tau})_+}+4\frac{(1-\hat{\tau})^2}{(1-\hat{\tau})_+}+4(1-\hat{\tau})^2\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+$$

and

$$-\frac{2}{\epsilon}\cdot 2\hat{\tau}(1-\hat{\tau})^{-1-2\epsilon}=\frac{2\hat{\tau}}{\epsilon^2}\delta(1-\hat{\tau})-\frac{4\hat{\tau}}{\epsilon}\frac{1}{(1-\hat{\tau})_+}+8\hat{\tau}\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+$$

Thus $\sigma^{NLO} = \frac{1}{16\pi^2}\left(\frac{4\pi\mu^2}{\hat{s}}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\left[\frac{1}{9}\,2g_w^2 g_s^2(1-\epsilon)\right]\times$

$$\left\{\frac{2}{\epsilon^2}\delta(1-\hat{\tau}) - \frac{4\hat{\tau}}{\epsilon}\frac{1}{(1-\hat{\tau})_+} + 8\hat{\tau}\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ - \frac{2}{\epsilon}\frac{(1-\hat{\tau})^2}{(1-\hat{\tau})_+} + 4\frac{(1-\hat{\tau})^2}{(1-\hat{\tau})_+} + 4(1-\hat{\tau})^2\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ + O(\epsilon)\right\}$$

$$= \frac{1}{16\pi^2}\left(\frac{4\pi\mu^2}{\hat{s}}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}\cdot\left[\frac{2}{9}g_w^2 g_s^2(1-\epsilon)\right]$$

$$\times\left\{\frac{2}{\epsilon^2}\delta(1-\hat{\tau}) - \frac{2}{\epsilon}\frac{1+\hat{\tau}^2}{(1-\hat{\tau})_+} + 4(1+\hat{\tau}^2)\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ + 4\frac{(1-\hat{\tau})^2}{(1-\hat{\tau})_+}\right\}$$

$$= \left(\frac{1}{9}g_s^2\right)2\tilde{\sigma}_0\cdot\frac{1}{16\pi^2}\left(\frac{4\pi\mu^2}{\hat{s}}\right)^{\epsilon}\frac{\Gamma(1-\epsilon)}{\Gamma(1-2\epsilon)}$$

$$\times\left\{\frac{2}{\epsilon^2}\delta(1-\hat{\tau}) - \frac{2}{\epsilon}\frac{1+\hat{\tau}^2}{(1-\hat{\tau})_+} + 4(1+\hat{\tau}^2)\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ + 4(1-\hat{\tau})\right\}$$

Eikonal Approximation at soft-Limit.

$$(\sigma^{NLO}_{soft}) = 2\tilde{\sigma}_0\frac{1}{16\pi^2}\left(\frac{4\pi\mu^2}{\hat{s}}\right)^{\epsilon_{IR}}\frac{\Gamma(1-\epsilon_{IR})}{\Gamma(1-2\epsilon_{IR})}\left\{\frac{2}{\epsilon_{IR}^2}\delta(1-\hat{\tau}) - \frac{4}{\epsilon_{IR}}\cdot\frac{1}{(1-\hat{\tau})_+} + 8\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+\right.$$

So we will see that when $\hat{\tau}=1$,

$$\sigma^{NLO}(\hat{\tau}=1) = \sigma^{NLO}_{soft}.$$

$\left(\hat{\tau} = \frac{M^2}{\hat{s}} = 1 \Rightarrow \hat{s} = M^2 \Rightarrow t = u = 0 \Rightarrow \text{soft photon}\right)$

~~Now let us sum over~~

5. Structure of the singularities at eikonal Approximation

I omit the common Factors here

$$(\sigma^{NLO}_{REAL}) = (\sigma^{NLO}_{coll})_{mixed} + (\sigma^{NLO}_{coll})_{unmixed} - (\sigma^{NLO}_{soft})$$

$$= \frac{2}{\epsilon^2}\delta(1-\hat{\tau}) - \frac{2}{\epsilon}\frac{1+\hat{\tau}^2}{(1-\hat{\tau})_+} + [8\hat{\tau} - 4(1-\hat{\tau})^2]\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ + 2(1-\hat{\tau})$$

The singular items of eikonal approximation is equal to the singular items of FULL real corrections, and only the finite items are different.

$$\sigma^{NLO}_{sing} = \sigma^{NLO}_{Virt} + \sigma^{NLO}_{Real}$$

$$= -\frac{3}{\epsilon_{IR}}\delta(1-\hat{\tau}) - \frac{2}{\epsilon}\frac{1+\hat{\tau}^2}{(1-\hat{\tau})_+} + 2(1-\hat{\tau})$$

$$+ \left[8\hat{\tau} - 4(1-\hat{\tau})^2\right]\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ - 7 - \frac{\pi^2}{3} + \delta_{scheme}$$

$$= -\frac{2}{\epsilon}\left(\frac{1+\hat{\tau}^2}{1-\hat{\tau}}\right)_+ + 2(1-\hat{\tau}) + \left[8\hat{\tau} - 4(1-\hat{\tau})^2\right]\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ - 7 - \frac{\pi^2}{3} + \delta_{scheme}$$

N11

In[6]:= $$\texttt{Simplify}\Big[\texttt{Series}\Big[\frac{\Gamma[1-2\epsilon]}{\Gamma[1-\epsilon]^2}\left(\frac{-1}{2\epsilon}\,\delta[1-\tau]+\frac{1}{(1-\tau)_+}-2\epsilon\left(\frac{\texttt{Log}[1-\tau]}{(1-\tau)}\right)_+\right)\mathcal{B}[-\epsilon,-\epsilon],\ \{\epsilon,0,1\}\Big]\Big]\ /\ \texttt{Log}[\tau]^{a}\_\ \delta[1-\tau]\to 0\ /\ \texttt{Log}[\tau]\,\delta[1-\tau]\to 0$$

Out[6]= $$\frac{\delta(1-\tau)}{\epsilon^2}-\frac{2}{(1-\tau)_+\,\epsilon}+4\left(\frac{\log(1-\tau)}{1-\tau}\right)_+ +O(\epsilon^2)$$

N12

In[7]:= $$\texttt{Simplify}\Big[\texttt{Series}\Big[\frac{\Gamma[1-2\epsilon]}{\Gamma[1-\epsilon]^2}\,\frac{1}{\epsilon}\,\mathcal{B}[-\epsilon,-\epsilon],\ \{\epsilon,0,1\}\Big]\Big]$$

Out[7]= $$-\frac{2}{\epsilon^2}+O(\epsilon^2)$$

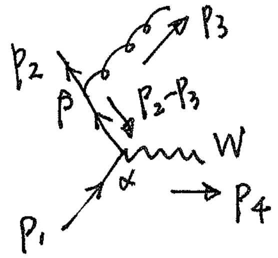
N13

In[8]:= $$\texttt{D1}=\texttt{Simplify}\Big[\texttt{Series}\Big[\frac{\Gamma[1-2\epsilon]}{\Gamma[1-\epsilon]^2}\left(\frac{-1}{2\epsilon}\,\delta[1-\tau]+\frac{1}{(1-\tau)_+}-2\epsilon\left(\frac{\texttt{Log}[1-\tau]}{1-\tau}\right)_+\right)(\mathcal{B}[-\epsilon,-\epsilon]-\mathcal{B}[1-\epsilon,-\epsilon]\,(1-\tau)),\ \{\epsilon,0,1\}\Big]\Big]\ /\ \texttt{Log}[\tau]^{a}\_\ \delta[1-\tau]\to 0\ /\ \texttt{Log}[\tau]\,\delta[1-\tau]\to 0$$

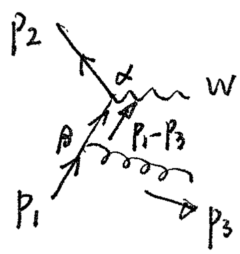
Out[8]= $$\frac{(\tau+1)\,\delta(1-\tau)}{2\epsilon^2}-\frac{\tau+1}{(1-\tau)_+\,\epsilon}+2(\tau+1)\left(\frac{\log(1-\tau)}{1-\tau}\right)_+ +O(\epsilon^2)$$

REAL EMISSION CORRECTIONS

1. Matrix Elements.



$$iM_1 = \bar{v}(p_2)\left[(-ig_s T^a_{ij}\gamma^\beta)\,\frac{-i(\not{p}_2-\not{p}_3)}{(p_2-p_3)^2+i\varepsilon}\,(i\Gamma^\alpha)\right]u(p_1)\cdot \epsilon^*_\alpha(p_4)\,\epsilon^*_\beta(p_3)$$



$$iM_2 = \bar{v}(p_2)\left[(i\Gamma^\alpha)\,\frac{i(\not{p}_1-\not{p}_3)}{(p_1-p_3)^2+i\varepsilon}\,(-ig_s T^a_{ij}\gamma^\beta)\right]u(p_1)\cdot \epsilon^*_\alpha(p_4)\,\epsilon^*_\beta(p_3)$$

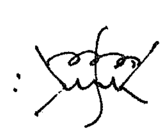
where. $\Gamma^\mu = \frac{g_W}{2\sqrt{2}}\gamma^\mu(1-\gamma_5)$

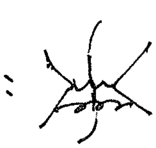
T^a : Gell-Mann color matrices

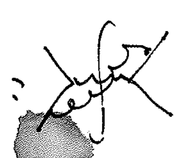
$\epsilon_\alpha(p_3), \epsilon_\beta(p_4)$: the W-boson and Gluon polarization vectors. respectively.

Squaring and summarizing over the spins and colors, we obtain

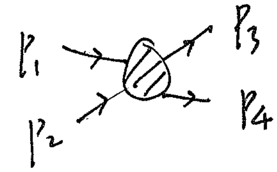
$$\sum|M|^2 = \frac{g_W^2}{8}\,g_s^2\,\mathrm{Tr}(T^aT^a)\cdot$$

$$\Big\{ \mathrm{Tr}[\not{p}_2\gamma^\beta(\not{p}_2-\not{p}_3)\gamma^\alpha\not{p}_1\gamma_\alpha(\not{p}_2-\not{p}_3)\gamma_\beta]\,\frac{1}{(2p_2\cdot p_3)^2}$$:  M_{11}

$$+\ \mathrm{Tr}[\not{p}_2\gamma^\alpha(\not{p}_1-\not{p}_3)\gamma^\beta\not{p}_1\gamma_\beta(\not{p}_1-\not{p}_3)\gamma_\alpha]\,\frac{1}{(2p_1\cdot p_3)^2}$$:  M_{22}

$$-\ 2\,\mathrm{Tr}[\not{p}_2\gamma^\beta(\not{p}_2-\not{p}_3)\gamma^\alpha\not{p}_1\gamma_\beta(\not{p}_1-\not{p}_3)\gamma_\alpha]\,\frac{1}{(2p_1\cdot p_3)(2p_2\cdot p_3)}\Big\}$$:  M_{12}

* Mandelstem Variables $(\hat{s}, \hat{t}, \hat{u})$:



$$\hat{s} = (p_1 + p_2)^2 = 2 p_1 \cdot p_2$$

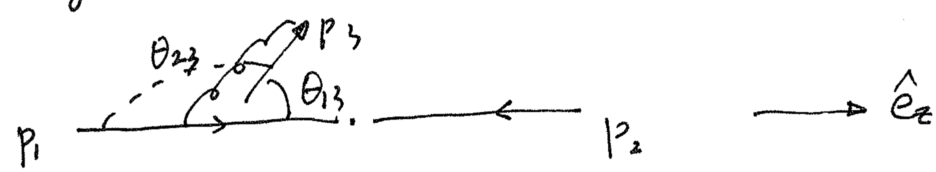
$$\hat{t} = (p_1 - p_3)^2 = -2 p_1 \cdot p_3$$

$$\hat{u} = (p_2 - p_3)^2 = -2 p_2 \cdot p_3$$

They satisfy the following relation.

$$\hat{s} + \hat{t} + \hat{u} = M_W^2$$

In the $q\bar{q}'$ C.M.F, we choose the incoming particles moving along z-axis.



then we have

$$p_1 = \left(\frac{\sqrt{\hat{s}}}{2}, 0, 0, \frac{\sqrt{\hat{s}}}{2}\right)$$

$$p_2 = \left(\frac{\sqrt{\hat{s}}}{2}, 0, 0, -\frac{\sqrt{\hat{s}}}{2}\right)$$

$$p_3 = (E_3, E_3 \sin\theta, 0, E_3 \cos\theta).$$

and

$$\hat{t} = -2 p_1 \cdot p_3 = -\sqrt{s}\, E_3 (1 - \cos\theta)$$

$$\hat{u} = -2 p_2 \cdot p_3 = -\sqrt{s}\, E_3 (1 + \cos\theta)$$

From relation $\hat{s}+\hat{t}+\hat{u}=M_W^2$, we know

$$E_3 = \frac{\hat{s}-M_W^2}{2\sqrt{\hat{s}}} .$$

So in the C.M.F. of the $q\bar{q}'$ system, the Mandelstem Variables could be rewritten as.

$$\hat{s} = 2p_1\cdot p_2$$

$$\hat{t} = -2p_1\cdot p_3 = -\hat{s}\left(1-\frac{M_W^2}{\hat{s}}\right)(1-y)$$

$$\hat{u} = -2p_2\cdot p_3 = -\hat{s}\left(1-\frac{M_W^2}{\hat{s}}\right)y$$

where.

$$y=\frac{1}{2}(1+\cos\theta) .$$

* γ- matrices.

We also need to treat the γ- matrices in the n-dim, $(n=4-2\varepsilon)$, which have the following properties:

$$\gamma^\mu \not{a}\gamma_\mu = -2(1-\varepsilon)\not{a}$$

$$\gamma^\mu \not{a}\not{b}\gamma_\mu = 4a\cdot b - 2\varepsilon\not{a}\not{b}$$

$$\gamma^\mu \not{a}\not{b}\not{c}\gamma_\mu = -2\not{c}\not{b}\not{a} + 2\varepsilon\not{a}\not{b}\not{c}$$

$$\mathrm{Tr}[\not{a}\not{b}\not{c}\not{d}] = 4\left[(a\cdot b)(c\cdot d)+(a\cdot d)(b\cdot c)-(a\cdot c)(b\cdot d)\right]$$

By using the above equation, we obtain the amplitude squares as following.

$$M_{11} = 16(1-\varepsilon)^2 \frac{\hat{t}}{\hat{u}} \qquad \sim \frac{1-y}{y}$$

$$M_{22} = 16(1-\varepsilon)^2 \frac{\hat{u}}{\hat{t}} \qquad \sim \frac{y}{1-y}$$

$$M_{12} = -32(1-\varepsilon) \frac{-\hat{s} M_W^2 + \varepsilon \hat{t} \hat{u}}{\hat{t}\,\hat{u}} \qquad \sim \frac{1}{y(1-y)}$$

* Collinear singularity.

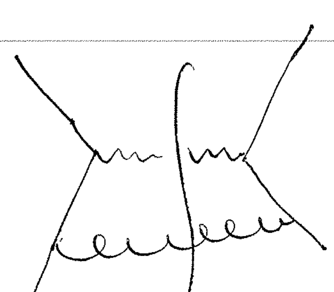
$y \to 0$. $\hat{u} \to 0$, $p_3 \parallel p_2$

$y \to 1$, $\hat{t} \to 0$, $p_3 \parallel p_1$.

* Soft singularity..

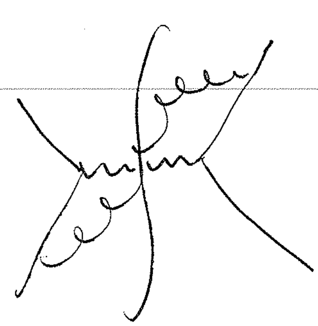
$$E_3 = \frac{\hat{s} - M_W^2}{2\sqrt{\hat{s}}}$$

$\hat{s} \to M_W^2 \Rightarrow E_3 \to 0 \Rightarrow \hat{u} \to 0$. and $\hat{t} \to 0$



$\sim \frac{1}{\varepsilon}$

collinear



$\sim \frac{1}{\varepsilon^2}$

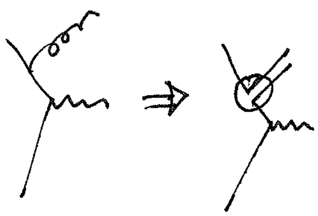
soft + collinear

Then

$$\sum |M|^2 = 16(1-\varepsilon)\left[(1-\varepsilon)\left(\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}}\right) + 2\frac{\hat{s}m_w^2}{\hat{t}\hat{u}} - 2\varepsilon\right]$$

The color factor

$$Tr(T^a T^a) = C_A C_F = 4$$



$\frac{1}{2}\delta_{ab} = 4$

Recalling that in n-dim

$$g_s \longrightarrow g_s \mu^{\varepsilon}$$

and averaging over the initial state spins and colors, we have the total amplitude square,

$$\sum \overline{|M|}^2 = \underbrace{\left(\frac{1}{2}\times\frac{1}{2}\right)}_{spin}\cdot\underbrace{\left(\frac{1}{3}\times\frac{1}{3}\right)\cdot 4}_{color}\cdot \mu^{2\varepsilon} g_s^2 \frac{g_w^2}{8}\, 16(1-\varepsilon)$$

$$\times \left[(1-\varepsilon)\left(\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}}\right) + 2\frac{\hat{s}M_w^2}{\hat{t}\hat{u}} - 2\varepsilon\right]$$

$$= \frac{2}{9}\mu^{2\varepsilon} g_w^2 g_s^2 (1-\varepsilon)\left[(1-\varepsilon)\left(\frac{\hat{t}}{\hat{u}} + \frac{\hat{u}}{\hat{t}}\right) + 2\frac{\hat{s}M_w^2}{\hat{t}\hat{u}} - 2\varepsilon\right]$$

$$= \frac{2}{9}\mu^{2\varepsilon} g_w^2 g_s^2 (1-\varepsilon)\left[(1-\varepsilon)\left(\frac{1-y}{y} + \frac{y}{1-y}\right) + \frac{2\hat{\tau}}{(1-\hat{\tau})^2}\frac{1}{y(1-y)} - 2\varepsilon\right]$$

where $\hat{\tau} = \dfrac{M_w^2}{\hat{s}}$.

2. TWO-BODY PHASE SPACE

$$\int dPS^{(2)} = \int \frac{d^{n-1}p_3}{2E_3(2\pi)^{n-1}} \cdot \frac{d^{n-1}p_4}{2E_4(2\pi)^{n-1}} \cdot (2\pi)^n \delta^{(n)}(p_1+p_2-p_3-p_4)$$

For on-shell final state particle.

$$\frac{d^{n-1}p_4}{2E_4(2\pi)^{n-1}} = \frac{1}{(2\pi)^{n-1}} d^n p_4 \, \delta^+(p_4^2 - M_W^2)$$

then

$$\int dPS^{(2)} = \int \frac{d^{n-1}p_3}{2E_3(2\pi)^{n-1}} \cdot \frac{1}{(2\pi)^{n-1}} d^n p_4 \, \delta^+(p_4^2 - M_W^2)\,(2\pi)^n \delta^{(n)}(p_1+p_2-p_3-p_4)$$

$$= \int \frac{d^{n-1}p_3}{2E_3(2\pi)^{n-1}} \cdot \frac{1}{(2\pi)^{n-1}} \delta[(p_1+p_2-p_3)^2 - M_W^2]$$

$$\doteq \frac{1}{(2\pi)^{n-2}} \int \frac{d^{n-1}p_3}{2E_3} \delta(s - 2E_3\sqrt{\hat{s}} - M_W^2) \qquad (\text{IN C.M.F. OF } qq')$$

Using polarcoordinates in $n-1$ dimensions.

$$d^{n-1}p_3 = |p_3|^{n-2} d|p_3| \, d\Omega_{n-2} = E_3^{n-2} dE_3 \, d\Omega_{n-2}$$

We obtain

$$\int dPS^{(2)} = \frac{1}{(2\pi)^{n-2}} \int dE_3 \frac{E_3^{n-3}}{2} \delta(s - 2E_3\sqrt{\hat{s}} - M_W^2) \int d\Omega_{n-2}$$

$$= \frac{1}{(2\pi)^{n-2}} \cdot \int dE_3 \cdot \frac{E_3^{n-3}}{4\sqrt{\hat{s}}} \delta\left(E_3 - \frac{\hat{s} - M_W^2}{2\sqrt{\hat{s}}}\right) \int d\Omega_{n-2}$$

$$= \frac{\pi^{\frac{n-2}{2}}}{(2\pi)^{n-2}\,\Gamma(\frac{n-2}{2})} \cdot \frac{1}{2\sqrt{\hat{s}}} \left(\frac{\hat{s} - M_W^2}{2\sqrt{\hat{s}}} \right)^{n-3} \int_0^{\pi} \sin^{n-3}\theta \, d\theta$$

Substituting $y = \frac{1+\cos\theta}{2}$,

$$\int dPS^{(2)} = \frac{\pi^{\frac{n-2}{2}}}{(2\pi)^{n-2}\,\Gamma(\frac{n-2}{2})} \frac{1}{2\sqrt{\hat{s}}} \left(\frac{\hat{s} - M_W^2}{2\sqrt{\hat{s}}} \right)^{n-3} \int_0^1 [4y(1-y)]^{\frac{n-4}{2}} 2dy$$

$$\overset{(n=4-2\varepsilon)}{=} \frac{1}{8\pi} \frac{(4\pi)^{\varepsilon}}{\Gamma(1-\varepsilon)} \frac{(\hat{s}-M_W^2)^{1-2\varepsilon}}{\hat{s}^{1-\varepsilon}} \int_0^1 dy \, [y(1-y)]^{-\varepsilon}$$

3. NLO Cross Section.

$$\sigma_{\gamma}^{NLO} = \frac{1}{2\hat{s}} \sum \overline{|M|^2} (P.S.^{(2)})$$

$$= \frac{1}{2\hat{s}} \cdot \frac{2}{9} \mu^{2\varepsilon} g_W^2 g_s^2 (1-\varepsilon) \cdot \frac{1}{8\pi} \frac{(4\pi)^{\varepsilon}}{\Gamma(1-\varepsilon)} \times \frac{(\hat{s}-M_W^2)^{1-2\varepsilon}}{\hat{s}^{1-\varepsilon}} \times$$
$$\int_0^1 dy \, [y(1-y)]^{-\varepsilon} \left[(1-\varepsilon)\left(\frac{1-y}{y} + \frac{y}{1-y}\right) + \frac{2\hat{\tau}}{(1-\hat{\tau})^2} \frac{1}{y(1-y)} - 2\varepsilon \right]$$

$$= \frac{2}{9} g_W^2 g_s^2 \frac{1}{2\hat{s}} \cdot \frac{1}{8\pi} \cdot \left(\frac{4\pi\mu^2}{\hat{s}}\right)^{\varepsilon} \frac{1}{\Gamma(1-\varepsilon)} \cdot (1-\hat{\tau})^{1-2\varepsilon} (1-\varepsilon)$$
$$\cdot \int_0^1 dy \, [y(1-y)]^{-\varepsilon} \left\{ (1-\varepsilon)\left(\frac{1-y}{y} + \frac{y}{1-y}\right) + \frac{2\hat{\tau}}{(1-\hat{\tau})^2} \frac{1}{y(1-y)} - 2\varepsilon \right\}$$

$$= \frac{2}{9} g_W^2 g_s^2 \frac{1}{2\hat{s}} \times \frac{1}{8\pi} \left(\frac{4\pi\mu^2}{M_W^2}\right)^{\varepsilon} \frac{1}{\Gamma(1-\varepsilon)} \cdot \hat{\tau}^{\varepsilon} (1-\hat{\tau})^{1-2\varepsilon} (1-\varepsilon)$$
$$\int_0^1 dy \, [y(1-y)]^{-\varepsilon} \left\{ (1-\varepsilon)\left(\frac{1-y}{y} + \frac{y}{1-y}\right) + \frac{2\hat{\tau}}{(1-\hat{\tau})^2} \frac{1}{y(1-y)} - 2\varepsilon \right\}$$

Using

$$\int_0^1 dy\, y^{\alpha}(1-y)^{\beta} = \frac{\Gamma(1+\alpha)\Gamma(1+\beta)}{\Gamma(2+\alpha+\beta)}.$$

$$\Gamma(x+1) = x\,\Gamma(x)$$

$$\Gamma(-\varepsilon) \sim -\frac{1}{\varepsilon}.$$

We obtain

$$\int_0^1 dy\,[y(1-y)]^{-\varepsilon}\frac{1}{y(1-y)} = \frac{\Gamma^2(-\varepsilon)}{\Gamma(-2\varepsilon)} = -\frac{2}{\varepsilon}\frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$\int_0^1 dy\,[y(1-y)]^{-\varepsilon} = \frac{\Gamma(1-\varepsilon)\Gamma(1-\varepsilon)}{\Gamma(2-2\varepsilon)} = \frac{1}{(1-2\varepsilon)}\frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$\int_0^1 dy\,[y(1-y)]^{-\varepsilon}\frac{y}{(1-y)} = \int_0^1 dy\,[y(1-y)]^{-\varepsilon}\left(\frac{1-y}{y}\right) = -\frac{1}{\varepsilon}\cdot\frac{1-\varepsilon}{1-2\varepsilon}\frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}.$$

then the y integral in the cross section yields

$$(1-\varepsilon)\left(-\frac{1}{\varepsilon}\right)\frac{1-\varepsilon}{1-2\varepsilon}\frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\times 2 + \frac{2\hat{\tau}}{(1-\hat{\tau})^2}\left(-\frac{2}{\varepsilon}\right)\frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)} - 2\varepsilon\frac{1}{(1-2\varepsilon)}\frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}$$

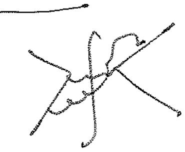
$$= \frac{\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\cdot\left\{-\frac{2}{\varepsilon} - 2\varepsilon - \frac{1}{\varepsilon}\frac{4\hat{\tau}}{(1-\hat{\tau})^2}\right\}.$$

$$\Rightarrow \sigma_r^{NLO} = \frac{2}{9}g_W^2 g_s^2\frac{1}{2\hat{s}}\cdot\frac{1}{8\pi}\left(\frac{4\pi\mu^2}{M_W^2}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\cdot x\,(1-\varepsilon)$$

$$\Big\{\underbrace{\hat{\tau}^{\varepsilon}(1-\hat{\tau})^{1-2\varepsilon}\cdot\left(-\frac{2}{\varepsilon}-2\varepsilon\right)} + \underbrace{\hat{\tau}^{1+\varepsilon}(1-\hat{\tau})^{-1-2\varepsilon}\cdot\left(-\frac{4}{\varepsilon}\right)}\Big\}$$

M_{11} 

M_{22} 

M_{12} 

* "plus" function.

$$\frac{1}{(1-z)^{1+2\varepsilon}} = -\frac{1}{2\varepsilon}\delta(1-z) + \frac{1}{(1-z)_+} - 2\varepsilon\left(\frac{\ln(1-z)}{1-z}\right)_+ + O(\varepsilon^2)$$

$$z^{\varepsilon}(1-z)^{-1-\varepsilon} = -\frac{1}{\varepsilon}\delta(1-z) + \frac{1}{(1-z)_+} - \varepsilon\left(\frac{\ln(1-z)}{1-z}\right)_+ + \varepsilon\frac{\ln z}{1-z} + O(\varepsilon^2)$$

Using "plus" function, we obtain

$$\sigma_{\gamma}^{NLO} = \frac{2}{9} g_W^2 g_s^2 \frac{1}{16\pi}\left(\frac{4\pi\mu^2}{M_W^2}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\cdot\frac{1}{\hat{s}}$$

$$\times\left[\frac{2}{\varepsilon^2}\delta(1-\hat{\tau}) - \frac{2}{\varepsilon}\frac{1+\hat{\tau}^2}{(1-\hat{\tau})_+} + 4(1+\hat{\tau}^2)\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ - 2\frac{1+\hat{\tau}^2}{1-\hat{\tau}}\ln\hat{\tau}\right]$$

$$= \frac{2}{9}(4\pi)^2\alpha\alpha_s\frac{1}{16\pi}\left(\frac{4\pi\mu^2}{M_W^2}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\frac{1}{\hat{s}}$$

$$\times\left[\frac{2}{\varepsilon^2}\delta(1-\hat{\tau}) - \frac{2}{\varepsilon}\frac{1+\hat{\tau}^2}{(1-\hat{\tau})_+} + 4(1+\hat{\tau}^2)\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ - 2\frac{1+\hat{\tau}^2}{1-\hat{\tau}}\ln\hat{\tau}\right]$$

$$= \frac{\pi^2}{3}\alpha\cdot\left(\frac{\alpha_s}{2\pi}\right)\frac{2}{3}\left(\frac{4\pi\mu^2}{M_W^2}\right)^{\varepsilon}\frac{\Gamma(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\frac{1}{\hat{s}}$$

$$\times\left[\frac{2}{\varepsilon^2}\delta(1-\hat{\tau}) - \frac{2}{\varepsilon}\frac{1+\hat{\tau}^2}{(1-\hat{\tau})_+} + 4(1+\hat{\tau}^2)\left(\frac{\ln(1-\hat{\tau})}{1-\hat{\tau}}\right)_+ - 2\ln\hat{\tau}\frac{1+\hat{\tau}^2}{1-\hat{\tau}}\right]$$