2. QED Lagrangian

(1) $\mathcal{L} = \bar{\psi}(i\not{D} - m)\psi - \frac{1}{4} F_{\mu\nu} F^{\mu\nu}$

$\not{D} \equiv \gamma_\mu D^\mu$, $\quad iD^\mu = i\partial^\mu - eA^\mu$

charge of e^- ($e = -|e|$)

$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

Dirac eq. $\quad (i\not{D} - m)\psi(x) = 0$

mass of e^-

Maxwell eq. $\quad \partial_\mu F^{\mu\nu} = e\bar{\psi}\gamma^\nu\psi$

$= e\, j^\nu$ current density

(2) Gaug Theory:

$\mathcal{L}$ is invariant under Gauge Transformation. ("local")

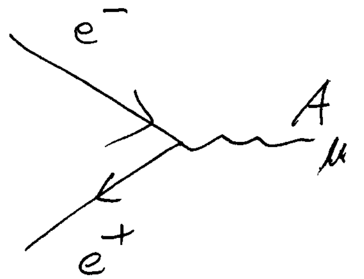
$\psi(x) \longrightarrow e^{i\alpha(x)}\psi(x)$, $\quad$ ($\alpha(x)$ depends on space-time x.)

$A_\mu(x) \longrightarrow A_\mu(x) - \frac{1}{e}\partial_\mu \alpha(x)$

3. Feynman Rules & Feynman Diagrams:

(1) Buiding blocks:

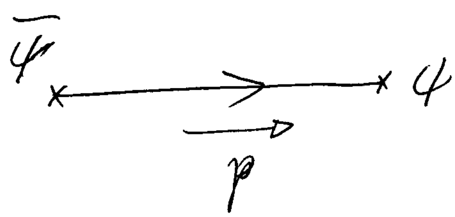
Vertex



$$\left(-ie\gamma_\mu\right)$$

Propagator

$$\frac{-i\, g_{\mu\nu}}{q^2 + i\varepsilon} \quad (\varepsilon \to 0^+)$$



$$\frac{i(\not{p}+m)}{p^2 - m^2 + i\varepsilon}$$

(where $p^2 \equiv p_\mu p^\mu$, $\not{p} \equiv \gamma_\mu p^\mu$)

plus external fermion (e^- and e^+)

and photon (A_μ)

Wavefunctions.

4. S-matrix and Cross Section

(1) Given $\mathcal{L}$, one can obtain $\mathcal{H}$, and then construct S-matrix element, which is denoted as $\mathcal{M}$.

scattering amplitude

(2) The probability is given by taking

$$\mathcal{M}^* \mathcal{M} = |\mathcal{M}|^2$$

Thus, the scattering cross section is

$$\frac{d\sigma}{d\Omega} = \left(\frac{\hbar}{8\pi M c}\right) \langle |\mathcal{M}|^2 \rangle$$

for an electron (mass m) scatters off a much heavier muon (mass $M \gg m$, otherwise, the same property as electron).

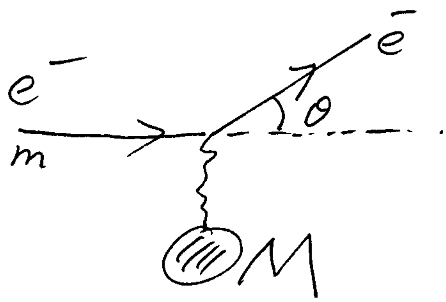
$\mathcal{M} \equiv$

$e^- \mu^- \longrightarrow e^- \mu^-$

$m = 0.511$ MeV

$M = 104$ MeV

In the rest frame of M,



$$d\Omega = d\cos\theta \, d\phi$$

$\Rightarrow$ $\frac{d\sigma}{d\Omega}$ is the differential cross section

After taking the non-relativistic limit ($v \ll c$), the QED prediction reduces to

(Mott formula)

Rutherford formula

$$\boxed{\frac{d\sigma}{d\Omega} = \left(\frac{\alpha \hbar c}{2 m v^2 \sin^2\left(\frac{\theta}{2}\right)} \right)^2}$$

where v is the speed of electron.

(Assuming that the recoil of M can be neglected.)

$\Rightarrow$ Measure $\alpha \equiv \frac{e^2}{\hbar c}$ (fine structure constant)